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**PASSIVE POSITIONING APPROACHES IN THE FUTURE**  
**POSITIONING SYSTEMS**

Master of Wireless Communication thesis

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Examiner and topic approved by the  
Faculty Council of the Faculty of  
Computing and Electrical Engineering  
on 1st March 2017

## ABSTRACT

**ELENA SERNA SANTIAGO:** Passive Positioning Approaches in the future positioning systems

Tampere University of Technology

Master of Wireless Communication thesis, 83 pages,

May 2017

Master's Degree Programme in Electrical Engineering

Major: Wireless Communications

Examiner: Assoc. Prof. Elena Simona Lohan

Keywords: WLAN, positioning, indoor environments, accuracy, distance-based, angle-based, IEEE, 802.11az, NGP

Wireless Local Area Networks (WLAN) are extending its fields of application and and, in addition, they start to support the indoor wireless positioning. Nevertheless, the challenges they have to deal with are highly compelling, since indoor environments are surrounded by walls, floors, obstacles, etc., that affect the signal propagation and, therefore, the accuracy of the positioning.

The thesis work addressed here aims to explore the features and the performance of a new positioning technique to be applied in indoor environments in future WLANs. The proposed technique consists of a passive positioning approach based on two well-known methods for positioning: distance-based and angle-based algorithms. Moreover, the used to standarize technique is developed on the basis of the new amendment of IEEE 802.11az in order to standarize the Next Generation Position Systems (NGP).

Therefore, the scope of the thesis work is mainly to verify whether the proposed passive positioning technique suits the requirements specified in 802.11az amendment through simulating different use cases.

A Matlab GUI interface has been created to test and compare various 802.11az-based scenarios. The main thesis findings are that, in order to support a positioning accuracy below 0.1m, timing measurements should have a maximum error variance of 80 ns and pure angle-based measurements are not sufficient.

## PREFACE

The thesis work has been carried out in the Department of Engineering of Electronics and Communications under the complete supervision of Associate Professor Elena Simona Lohan. She gave me the opportunity to develop the topic of this thesis, where I have been able to learn new concepts about indoor positioning systems as well as know the state of the art of these technologies. I would like to thank Simona for the dedication and the support that I have received from her, helping me with the development of the thesis, allowing me have periodically meetings, as well as revising every detail of my work and guiding me to next steps in order to finish successfully the Master's Thesis.

This thesis work represents the ending of a lovely story, my exchange period in a magical place: Finland. Therefore, I would like to thank you to all the people that have met for the moments lived and supporting me during these last months, spetial mention to my closest friends, for all the moments we shared that I will always keep on my heart.

Finally, I would like to dedicate this thesis to my family for having always been by my side, supporting me to achieve my goals, in special...

*... To my parents for teaching me to be a better person everyday, to support myself to achieve my goals and surpass myself, but above all, to love me unconditionally: You are the reason of my success!*

*... To my brothers for always taking care of me.*

*... To my aunt and uncle who are my second family for encouraging and trusting me unconditionally.*

*... To my love for believing always in me.*

Tampere, 24.05.2017

Elena Serna Santiago

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## ABBREVIATIONS

AP	Access Point
AOA	Angle Of Arrival
ACK	Acknowledgment
BPSK	Binary Phase Shift Keying
BSS	Basic Service Sets
BSA	Basic Service Area
BCC	Base Station Colour Code
BSSID	Basic Service Set Identifier
BSIC	Basic Station Identity Code
CR	Cognitive Radio
DBCA	Dynamic Bandwidth Channel Access
FPS	Future Positioning Systems
FTM	Fine Timing Measurement
FFT	Fast Fourier Transform
GPS	Global Positioning Systems
GNSS	Global Navigation Satellite Systems
GUI	Graphical User Interface
GSM	Global System for Mobile Communications
HD	High Definition
HEW	High-Efficiency WLAN
HW	Hardware
IEEE	Institute of Electrical and Electronics Engineers
IoT	Internet of Things
LoS	Line of Sighth
LMA	Levenberg-Marquadt Algorithm
MS	Mobile Station
MIMO	Multiple Input Multiple Output
MU-MIMO	Multi-User Multiple Input Multiple Output
MAC	Medium Access Layer
NIC	Network Interface Card
NGP	Next Generation Positioning Systems
NLoS	Non Line of Sighth
NCC	Network Colour Code

N-R	Newton-Raphson method
$N_{AP}$	Number of APs
QPSK	Quadrature Phase Shift Keying
OFDM	Orthogonal Frequency Division Multiplex
OBSS	Overlapping Basic Service Sets
OFDMA	Orthogonal Frequency Division Multiple Access
PHY	Physical Layer
QAM	Quadrature Amplitude Modulation
RAW	Restricted Access Window
RTT	Round Time Trip
RSS	Received Signal Strength
RSSI	Receive Signal Streghth Indicator
RF	Radio Frequency
STA	Station
SDMA	Spatial Diversity Multiple Access
SC-OFDMA	Single Carrier Orthogonal Frequency Division Multiple Access
TUT	Tampere University of Technology
TWT	Target Wake Time
TOA	Time Of Arrival
TDOA	Time Difference Of Arrival
TIM	Traffice Indication Map
TOD	Time Of Departure
UHF	Ultra High Frequency
VHF	Very High Frequency
WLAN	Wireless Local Area Network
2D	2-Dimensions
3D	3-Dimensions

## SYMBOLS

$\alpha_i$	azimuth angle with respect $AP_i$
$c$	speed of light
$d_{mvl,AP_i}$	distance between mobile and $AP_i$
$f$	Objective function to minimize
$J$	Jacobian
$\lambda$	Step of Gauss-Newton method
$\mu_t$	Mean value of the noise in time estimation
$\mu_\alpha$	Mean value of the noise in azimuth estimation
$\mu_\theta$	Mean value of the noise in elevation estimation
$N(\mu, \sigma^2)$	Noise with mean value $\mu$ and variance $\sigma^2$
$\nabla$	Gradient
$P_{RX}$	Received Power
$P_{TX}$	Transmitted Power
$\sigma_t$	Time variance
$\sigma_\alpha$	Azimuth variance
$\sigma_\theta$	Elevation variance
$t_s$	time of start
$t_f$	forwarding time
$t_i$	time of arrival from $AP_i$
$t_0$	emitting time
$\theta_i$	theta angle with respect $AP_i$
$x_i$	$x_i, y_i, z_i$ coordinates of the $AP_i$
$x$	x,y,z coordinates of the mobile
$x_0$	Initial guess vector (x,y,z)
$x_{n+1}$	Next initial guess vector (x,y,z)
$y_{obs}$	Time, azimuth and angle observables

# 1. INTRODUCTION

The Wireless Local Area Network (WLAN) is increasingly gaining importance nowadays. This widespread infrastructure offers multiple services that make our daily life easier. Among these services, WLAN offers the possibility to locate devices economically, making these networks interesting to study.

WLAN positioning can be applied both in indoor and outdoor environments. However, its main operating field is in-buildings, because in outdoor environments Global Positioning Systems (GPS), and nowadays also other Global Navigation Satellite Systems (GNSS), remain the dominant technologies, due to their high accuracy and global coverage.

Indoor positioning via WLAN infrastructure presents multiple advantages that make it more suitable than cellular-based or GNSS for indoor positioning. Some of these advantages are: higher accuracy compared with cellular-based and an existing infrastructure supporting indoor location compared with GNSS, whose availability indoors remain quite limited [2] [3].

The fact that WLAN positioning uses an infrastructure already deployed is really useful and interesting, not only from the user point of view, but also from the network providers point of view, because it involves a low-cost service. Also for the location-based service providers such a positioning technology is of interest, where applications such as assistant navigation in stores/ stadiums or finding lost devices are becoming more common to give them market value and they only need an operating WLAN to work.

This new market trend means that the future WLANs will have to support mechanisms and tools to provide users in-building location and tracking properly with exceptional accuracy, immediate response, low cost and preferable no additional software for client devices.

Therefore, IEEE 802.11 is working on a new standard called "Next Generation Positioning, 802.11az", which will enhance the indoor positioning by focusing on several key concepts, such as [4] [5] [6]:

- **High accuracy positioning.** The adoption of the technology brings requirements such as centimeter accuracy for the new age user experience [4]. Thus, 802.11az aims to improve the accuracy of previous contributions to micro location, and to reach roughly an accuracy of the cm order (e.g., 0.1m). However, current technology has a limitation of 0.7 m accuracy being 2-4m in real scenarios [4], thereby 802.11az has to face a challenge to get an improved accuracy.
- **Direction finding and tracking.** Some of the new usage cases are based on navigation in public buildings, indoor locations maps and directions. These cases require the new positioning algorithm implements mechanisms allowing the direction finding and tracking.
- **Scalability and reducing overhead.** In previous standards such as 802.11ac and 802.11AS-2011, the Received Signal Strength Indicator (RSSI) and the Fine Timing Measurement (FTM) were defined as suitable positioning algorithm in WLAN. Nevertheless, these algorithms present some drawbacks nowadays, such as:
  - RSSI is a time-consuming protocol and susceptible to environmental changes [7], which make it not suitable in the new set of usage models we have described where the user is continuously moving and the network has to estimate every time the position of the mobile.
  - FTM overloads the network due to the number of messages sent to locate the mobile: a minimum of 6 messages are required per single fix per AP and 4 APs are required per single fix [4]. In dense environments, the use of this protocol is unfeasible and a broadcast protocol will be more suitable because it will reduce the overhead of the network.

To mitigate all these key challenges, it is necessary to develop an algorithm based on an infrastructure broadcast protocol that reduces overhead, improves scalability and accuracy.

Thus, IEEE has defined 802.11az as a protocol to support passive positioning algorithm in which the stations (STAs) are sending periodically frames with their position information and other network parameters that allow the mobile to estimate its position at every moment. The IEEE 802.11az has been still under study since September of 2015. It is expected to be standardized in a first release at the end of 2017 and the final version in 2020 [8].

Although there are still some unknowns about this new amendment, the TGaz working group is reporting almost monthly some of the functionalities and requirements of the Next Generation Positioning (NGP) systems [8]. At this moment, the functional requirement document has been initiated, as well as the approved accuracy/coverage and millimetric wave positioning requirements.

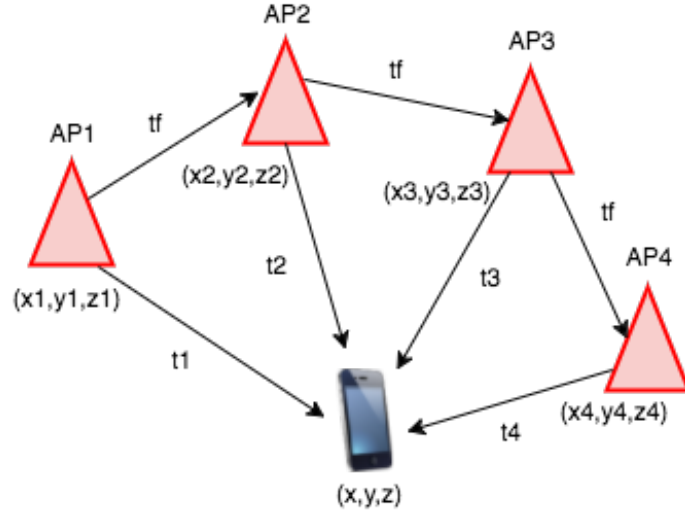
## 1.1 Thesis Objectives

NGP systems are being built in such a way to enhance the current positioning techniques based on WLAN. However they have to deal with some new challenges, which are difficult to fulfil, before putting these systems into the market. Thereby, it is necessary to start to work on it, proposing different algorithms, mechanisms or techniques that help to reach these objectives.

The thesis work considered here is a research work following the idea mentioned above. The main objective of the thesis has been to study and develop an algorithm that allows to positioning mobile devices without the need for the mobile to send information to the network, i.e., in a passive way, based on the existing specifications of 802.11az standard.

The idea is to have a network formed by several Access Points (AP), which will be sending to the mobile through the network some frames with different identification parameters, which will allow to the mobile estimates its position. The identification parameters are the following:

- **Position of the AP:**  $x_i, y_i, z_i$  coordinates of the considered AP.
- **Time of start,  $t_s$ :** reference time from the first AP.
- **Forwarding time,  $t_f$ :** it is a constant time equal for all AP that establishes the existing time between AP.



**Figure 1.1** Network Infrastructure

The above-mentioned idea is illustrated schematically in Figure 1.1 where it can be identified:

- APi: Access Point identifier.
- $x_i, y_i, z_i$  : AP i coordinates.
- x,y,z: mobile coordinates.
- $t_f$ : forwarding time.
- $t_i$  : time of arrival of the signal from APi.
- $t_s$  : starting time of transmission from the first AP.

As it can be seen in Figure 1.1, the APs are sending periodically frames to the user and the mobile will use the parameters of the frames of some APs to estimate its position.

The number of the AP considered will depend on the environment (e.g., building size if indoor positioning or urban density if outdoor positioning). However, the mobile will not need all the APs to estimate its position accuracy, it will be enough using just some of APs.



Thus, one of the things done during this thesis has been to study how many AP are required to estimate the position of the mobile accurately, considering the objective error that the IEEE has defined for this standard (roughly 0.1m [4]).

## 1.2 Author's Contributions

The contributions of the author to the thesis have been mainly:

- Study and identify the requirements of the Next Generation Positioning Systems (NGP) for different uses cases.
- Define and develop in Matlab a new positioning algorithm to support NGP and its requirements, characterizing its parameters.
- Create an user interface (GUI) in order to test the functionalities of the proposed algorithms for NGP system under different study cases.
- Testing and comparing time of flight and angle of arrival-based estimates in various scenarios.

Nevertheless, the author has focused along this thesis work on studying the accuracy of the estimation position with the proposed technique, under different assumptions that try to represent real cases, such as:

1. **Study of the influence of the noise in the accuracy of the estimated position.** Through this case, we aimed at getting an idea about how noise affect to the measurement of the mobile position. The thesis aimed to answer questions such as: how many AP are needed in presence of noise? Is it possible to estimate the position having any noise? How does the noise affect to all the estimators?
2. **Study of the influence of the LoS and NLoS environments.** This case has a huge importance in all systems based on WLAN. As we know, according to the environment in which we are placed, the systems can work properly, regularly or directly do not work, depending on the number of multipaths. In this way, we studied how the estimation is in LoS and NLoS environments, getting as well an idea about how the multipaths affect to the estimators and

to the number of the AP necessary in the network to fulfil the requirements that the standard has.

3. **Creating a Graphical User Interface (GUI) to support previous use cases.** We have built a GUI in Matlab to set-up the different scenarios under studying. This GUI will allow us to evaluate the functionalities of the 802.11az standard, through the obtained graphs and tables.

The methodology followed in this thesis to study the previous cases, consisted of four main parts:

1. Gathering documentation about the standard, functionalities and requirements.
2. Defining uses cases and the corresponding functions in Matlab.
3. Building a GUI interface for 802.11az-based positioning.
4. Running simulations under various scenarios and parameters in order to find the answers to the above-mentioned questions.

## 1.3 Thesis Structure

In the following chapters, these cases will be explained deeply. In the first place, the “Passive Positioning” concept will be introduced as the basis of the standard 802.11az, as well as some passive positioning algorithms possible to use in 802.11az amendment. Secondly, the latest WLAN standards until 802.11az will be briefly described, in order to show the evolution of the standards due to the adoption of the technology. Moreover, along Section 3.7 it will be introduced the key challenges 802.11az standard will have to face. The simulator developed to test the work depicted in the thesis will be described in Section 5, showing some of the results obtained and comparing the proposed method with other positioning techniques based on angle estimation. Finally, in Chapter 6 will be summarized the main findings of the thesis work addressed.

## 2. PASSIVE POSITIONING CONCEPT

### 2.1 Dual definition of "Passivity"

Until now, indoor location systems have been focused on device-positioning using only active devices, i.e., the device needs to send to the network some information to positioning. Recently, a new alternative trend is emerging in wireless environments, called "Passive Positioning".

Passive positioning is not exactly a new concept due to the fact it is commonly implemented and used in outdoor positioning systems such as GNSS. However, passive positioning terminology can be used in two different and opposite contexts:

1. The network does not have any knowledge about the user's position. The user is the only responsible of calculating his/her position and the only one who will have that information. The user terminal is thus 'passive' in the sense that it does not need to send any location-information back to the network and can do the positioning by itself, in a fully mobile-centric mode [9].
2. The network locates and tracks the user without his/her authorization. The user terminal can be also seen as 'passive' in the sense that the user does not take an active part in the localization process [10].

The thesis work depicted here is about passive positioning definition 1, and the main idea of this definition is the user terminal acts as a passive device: it receives signalling information from the network, but it does not send back information that might enable the network to locate it.

This method of positioning has special importance to the network because it will get to reduce the overhead of the network and simplify the calculations in the position of the mobile estimation, making the positioning process easier. In addition, this

method will be very important for the user's privacy point of view, since the passive positioning approach does not divulge any unauthorized information from the user.

In the traditional active positioning systems, every time the user wants to go to a specific place or get a direction, he or she needs to interact with the network in order to get that information. The work principle in an active positioning concept (according to the terminology used in our thesis) basically consists of the following:

- The user sends a request to an AP to get the desired route. In this request, the user is not only communicating to the AP the final place he or she wants to go, he/she sends some information that allows the network to estimate the position of the mobile. Sometimes, one of the vector parameters sent is the mobile's position. In this way, the network already knows the position of the mobile device and can locate and track it easily.
- The next step, in case the position of the mobile has not been sent, is the estimation of the mobile position by the AP. Once it has been done, the AP queries a server in order to get the route requested by the user.
- Finally, the AP sends the route obtained from the server back to the mobile.

Therefore, active positioning systems are usually called "network based positioning" where the APs receive signals from the mobile stations and the position determination is carried out remotely on a server within the network [11].

On the other hand, the active positioning gives to the operators, and also to some service providers, full knowledge about the position of each user connected to the network. The users cannot control their positioning privacy, and this might be an important drawback for them. Moreover, malicious attacks on the network may make user more vulnerable to location thefts or location-based frauds.

In that way, passive positioning systems (in the terminology we adopted in this thesis) are presented to satisfy privacy issues and to increase the user control, since they do not require any forwarding of information to the network.

The estimation of the position using passive positioning method is not trivial at all because, as it has been mentioned, the mobile does not send any information of its position to the network. The network is the only one who sends information and the mobile device is the responsible to estimate its position using this information.

Let's consider a user scenario as follows: the user is in a shopping mall and he/she wants to go to a specific direction, it is not required that he/she sends a request to the network asking for the navigation. With the passive positioning method, at the beginning of the entrance to the store, a database containing all the possible routes to all the possible places will be downloaded automatically in the mobile device. Thus, the network does not know which it is the position of the user, because he/she has not shared it. The user will just specify the desired destination on the database and the navigation route will start.

The work principle of the passive positioning method is the following:

- The APs are continuously sending information to the users via broadcast messages. The sent frames containing, among other parameters, the position of each AP in the building where user is placed.
- The users receive all the information sent by the network, and with the help of some algorithms incorporated in the mobile devices, they will be able to estimate their position.
- Once the position has been estimated, the database takes this value and according to the desired destination it loads the navigation map.

Passive positioning can be also seen as “mobile based technique”, because the determination of the mobile position is exclusively carried out at the MS using the signals received from the APs.

Comparing this method with the previous one, we can notice that the environment type and the positioning algorithms have strong impact on the achievable accuracy.

## 2.2 Positioning Algorithms

There are several algorithms can be used in passive positioning systems to estimate the position of the mobile. They are usually classified according to the parameter measured to positioning. The most widely used are the following:

- Based on time propagation measurement: Time of Arrival (TOA), Time Difference of Arrival (TDOA) and Round Time Trip (RTT).

- Based on angle propagation measurement: Angle of Arrival (AOA).
- Based on Received Signal Strength (RSS) measurements.

The TOA, TDOA and AOA algorithms are briefly described in this chapter, while the RTT and RSS algorithms, as well as other algorithms, less used currently but which can be specific to 802.11az, will be described in Chapter 4.

### 2.2.1 Time of Arrival (TOA)

The time of arrival estimation principle consists of measuring the signal propagation time between the transmitter and the receiver and calculating the distance multiplying that time by the speed of the light.

In order to measure correctly the distance between the transmitter and the receiver, it is necessary to know the emitting time,  $t_0$ . Thus, the APs involved in the TOA measurements will have a common timing reference, with a known mobile transmission time [12].

The TOA measurement defines a centered circle at the AP and radius the distance between the mobile and the AP. To locate the user with enough accuracy and avoiding ambiguities, at least three AP will be needed. The position of the mobile will be given by the intersection of these three circles as it is illustrated in Figure 2.1.

The position of the mobile is then calculated applying the following relation:

$$\|x - x_i\| = c(t_i - t_0), i = 1, 2, 3 \dots \quad (2.1)$$

where:

$x$  = mobile coordinates vector( $x, y, z$ )

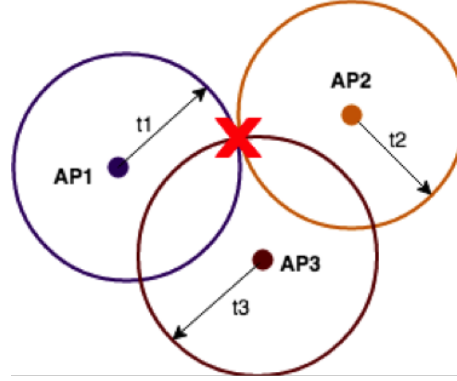
$x_i = AP_i$  coordinates vector( $x_i, y_i, z_i$ )

$c$  = speed of light,  $3 \cdot 10^8 m/s$

$t_i$  = time of arrival of incoming signal from  $AP_i$

$t_0$  = emitting time

This method has several advantages and disadvantages that make necessary to study firstly its convenience according to the use case.



**Figure 2.1** Time of Arrival (TOA) principle

The most important advantages of the TOA algorithm are its high accuracy and the ease of use of itself. As it can be seen in the Figure 2.1, it is just necessary to have three APs in order to position the user univocally. Moreover, TOA does not require to implement complex hardware in the APs to run it, which is quite interesting when aiming low cost implementations.

However, TOA has some drawbacks that are important to point out. In the first place, to estimate correctly the position of the Mobile Station, all the clocks (MS and AP) need to be synchronized. Unfortunately, in most cases involving WLANs, the MS clock is not synchronized with the AP clock, producing errors in the measurements similar to the noise effects [13].

In the presence of noise due to the unsynchronized clocks or because the environment is noisy, the TOA measurement is highly inaccurate. The resulting the three circles do not intersect at a single point and this is causing important errors in the estimation.

Moreover, TOA algorithm is only valid in LoS environments where there is not any obstacle between the AP and the MS allowing estimate the position of the user [13]. In NLoS environments the estimation position will be erroneous due to the number of multipaths can be found. Although TOA can be applied in NLoS environments, there is not possible to guarantee to work properly.

In addition, in order to compute the ToA, it is needed to know the signal modulation

type and carrier frequency, and thus the receiver should be tuned to the transmitter waveform. It also means that we cannot build generic TOA algorithms, but each of them has to be designed according to the type of the waveforms and frequency bands used at the transmitter side.

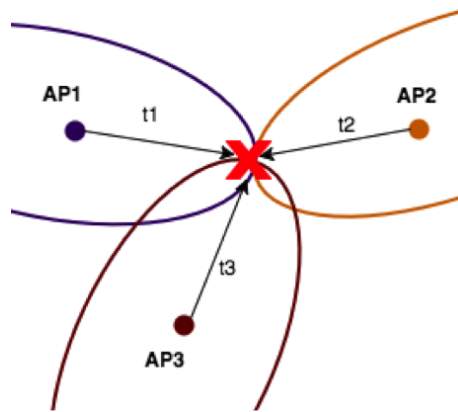
### 2.2.2 Time Difference of Arrival (TDOA)

The TDOA is an algorithm similar to the TOA. The TDOA algorithm estimates the position of the mobile by measuring the time difference between two arrival signals, typically from at least two AP.

The TDOA method can be considered as an enhanced version of the TOA algorithm. Unlike the TOA, the TDOA does not require the perfect synchronization between the clocks of the AP and the MS. TDOA just requires that the clocks of each AP to be synchronized. Then, it is not necessary to know the exact emitting time as before [11].

This improvement makes the TDOA algorithm more accurate than the TOA, since in this case there would not be timing inaccuracies that make difficult the estimation process.

Since TDOA method estimates the position of the mobile by measuring the time difference of arrival of the two AP signals, the TDOA algorithm is usually called “hyperbolic positioning”. The measurements locate the receiver on a hyperbola with two transmitters as a foci [13], as it is illustrated in Figure 2.2.



**Figure 2.2** Time Difference of Arrival (TDOA) principle



The equation to determine the target position is the following:

$$||x - x_i|| - ||x - x_j|| = c(t_i - t_j), \quad i, j = 1, 2, 3 \dots j \neq i \quad (2.2)$$

where:

$x$  = mobile coordinates vector( $x, y, z$ )

$x_i = AP_i$  coordinates vector( $x_i, y_i, z_i$ )

$x_j = AP_j$  coordinates vector( $x_j, y_j, z_j$ )

$c$  = speed of light,  $3 \cdot 10^8 m/s$

$t_i$  = time of arrival of incoming signal from  $AP_i$

$t_k$  = time of arrival of incoming signal from  $AP_j$

As it can be seen in Figure 2.2, with TDOA algorithm, also, at least three AP are needed to estimate the position of the mobile in two-dimensional coordinates (2D), and at least four AP to estimate it in three-dimensional coordinates (3D).

Although this method can be considered as an enhancement of the TOA and gives us higher accuracy in the estimation process, it has also some drawbacks. As it in the previous case, the TDOA algorithm is also affected by the environment (multipath propagation), noise effects and interference. Therefore, it is preferable to use it in line of sight environments such as open space or in large open buildings [14], where the correct functioning of the algorithm can be guaranteed.

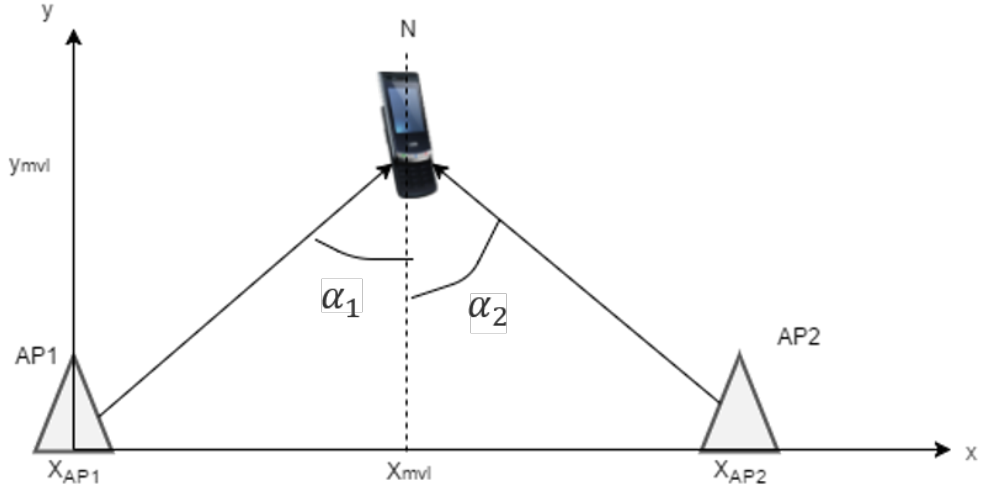
### 2.2.3 Angle of Arrival (AOA)

In addition to the previous methods, WLAN positioning based in angle measurement is commonly used as well to estimate the position of the mobile devices.

The Angle of Arrival (AOA) algorithm consists of measuring the angle of arrival between the line that runs from the AP to the MS and a line from the AP with a predefined direction chosen as a reference direction, normally pointing to the North [14].

In order to measure the position of the mobile device, at least two APs at known

positions will be needed. The position will be calculated using simple geometry: the target position will be determined by the cross of the two lines from the APs. This principle is illustrated in Figure 2.3.



**Figure 2.3** Angle Of Arrival (AOA) 2D

The Figure 2.3 illustrates the basic principle of the AOA in 2D passive positioning environment. The mobile determines the angle of arrival from each AP using simple trigonometric, as follows:

$$\alpha_1 = \text{atan2} \left| \frac{x_{mvl} - x_{AP1}}{y_{mvl} - y_{AP1}} \right| \quad (2.3)$$

$$\alpha_2 = \text{atan2} \left| \frac{x_{mvl} - x_{AP2}}{y_{mvl} - y_{AP2}} \right| \quad (2.4)$$

Where:

$x_{mvl}$  = x-coordinate of mobile

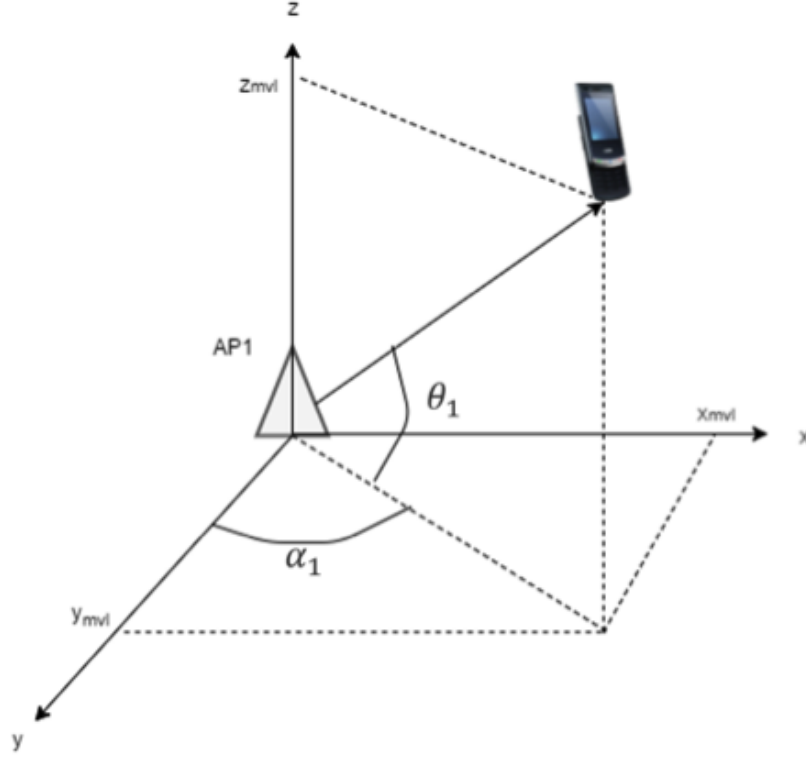
$x_{AP1}$  = x-coordinate of  $AP_1$

$x_{AP2}$  = x-coordinate of  $AP_2$

$y_{AP1}$  = y-coordinate of  $AP_1$

$y_{AP2}$  = y-coordinate of  $AP_2$

The topic addressed in this thesis considers the 3D-positioning, thereby besides the azimuth angle, the elevation needs to be estimated in order to get the value the coordinate  $z$  of the mobile. The AOA algorithm is illustrated in Figure 2.4.



**Figure 2.4** Angle Of Arrival (AOA) 3D

The azimuth angle  $\alpha_1$  is still estimated with formula 2.3 for the 3D positioning, and the elevation angle depends on the  $z$  coordinate of the mobile as shown in equation 5.3.

$$\theta_1 = \text{asin} \left| \frac{z_{mvl} - z_{AP1}}{d_{mvl,AP1}} \right| \quad (2.5)$$

$$d_{mvl,AP1} = \sqrt{(x_{mvl} - x_{AP1})^2 + (y_{mvl} - y_{AP1})^2 + (z_{mvl} - z_{AP1})^2} \quad (2.6)$$

where:

$z_{mvl}$  = z-coordinate of mobile

$z_{AP1}$  = z-coordinate of  $AP_1$

$$d_{mvl,AP1} = \text{distance between mobile and } AP_1$$

For 3D estimates, the azimuth and elevation angles from at least 3 APs are needed to be measured, in order to be able to estimate the (x,y,z) coordinates of the mobile

As it has been mentioned, the mobile device is responsible of measuring the angles of arrival,  $\alpha$ ,  $\theta$ , applying geometric relations as it is shown in Figures 2.3 and 2.4. The position of the mobile is in turn estimated using the equations (2.3) and (5.3) to signals received from all APs.

The AOA algorithm offers a practical alternative to positioning based on time measurement such as presented in sections 2.2.1 and 2.2.2, because it does not require time synchronization. Thus, this is an advantage and useful to consider in environments in which it is not possible to get the clocks synchronization.

However, this algorithm has certain drawbacks. In the first place, AOA is an expensive algorithm that requires more equipment compared with the TOA or TDOA. The main reason is that the receiver has to have directional antennas or antenna arrays in order to determine the incoming angle. Otherwise, the position of the user will be estimated with high errors.

Furthermore, the angles are much harder to measure than the distance from the AP to the MS. This implies the AOA is a method which demands more energy. In that way, the AOA is the most expensive algorithms of the presented in the chapter because it requires special antennas, hardware modification and also it consumes more energy [14].

On the other hand, the accuracy of the AOA is affected by the accuracy of the antennas used in the network. As it has been mentioned, the AOA algorithm requires special antennas in order to obtain the position through the angle estimation of the coming signals. The accuracy is closely related to the spatial resolution and the directionality of these antennas, thus in that cases in which the installed antennas do not have good characteristics, the accuracy will decrease heavily [15].

Moreover, the AOA method is susceptible enough to the environments in which there is not LoS between the AP and MS, since any obstacle between both sides will affect drastically to the estimation [15].

### 3. NEW STANDARDIZATION EFFORTS IN WLAN SYSTEMS

Due to the constant advances in technology, the needs of the users are continuously changing, thus the systems need to be updated with the latest available technology. In such a way that, there is an organization, known as IEEE, which is responsible to promote the integration and apply these advances in technology for the benefit of the society, defining the operation rules for the systems to guarantee the correct working and the harmony among different systems and technologies.

Since its creation, IEEE has standardized a huge number of amendments related with the latest technology available in all the technical fields: information technology, electronics and science. Thereby, IEEE has different groups of work and standards according to the considered field.

Due to the topic depicted in this thesis, the standard of interest for us is the IEEE 802.11. This standard comprises all the operation rules for a Wireless Local Area Network (WLAN) and it is formed by different versions according to the technology available in that moment. Exactly, since the first standard in 1997 (802.11-1997) IEEE has defined a total of 42 standards. In Table 3.1 the main 802.11\* standards defining physical layer features are cited [16]:

In the following points the newest 802.11 standards, from 2013 onwards, will be described.

#### 3.1 WLAN 802.11ac standard

The previous standards for Wireless Local Area Networks (WLANs) developed from 1997 were designed to connect PC in the home, office, and to allow connectivity “on the road” [17]. However, the needs changed and new usage models were needed to be covered. These new usage models would require mainly higher throughput but

**Table 3.1** WLAN standards and amendments

Amendment	Date of creation	Brief Description
802.11a	1999	WLAN operating in 5GHz ISM band with data rates up to 54 Mbps
802.11b	1999	WLAN operating in 2.4 GHz ISM band with data rates up to 11 Mbps
802.11g	2003	Enhancement of 802.11b, with data rates up to 54 Mbps
802.11n	2009	Enhance of the combination of 802.11g and 802.11a. WLAN operating in 2.4 and 5 GHz with data rates up to 600 Mbps
802.11ac	2013	WLAN operating in frequency bands < 6 GHz with data rates up to 1 Gbps
802.11ad	2013	Very high throughput at 60 GHz
802.11af	2014	White-Fi, Wi-Fi in TV spectrum white spaces
802.11ah	2016	Supporting Internet of Things (IoT). Wi-Fi using frequency bands below 1 GHz
802.11ax	Started in 2014	High Efficiency amendment in ultra dense environments working at 2.4 and 5 GHz
802.11ay	Started in 2015	High speed multi-gigabit WiFi at 60 GHz
802.11az	Started in 2015	Future positioning systems in WLAN

also rapid upload/download of files as video contents.

In order to deal with these challenges, the standard 802.11ac was defined. This standard is also known as “WIFI Gigabit” and it allows very high-throughput at frequencies below 6GHz. With this standard, it is intended to achieve multi-user ultra-high-rates in WLAN, providing data rates up to 1 Gbps (maximum 7 Gbps, with the higher configuration) in the frequency band of 5 GHz. This target is ten times more than the data rate of the previous standard 802.11n.

Besides the high data rate allowed with this standard, IEEE 802.11ac allows to use more bandwidth due to the fact it adds channel bandwidths of 80 MHz and 160 MHz with both contiguous and non-contiguous 160 MHz channels for flexible channel assignment [18]. Moreover, it is allowed to bond more than one channel to increase the capacity in that cases which are needed. This more channel bonding provides speed increases 177 and 333 percent, respectively [19], compared to the previous standards.

Moreover, it adds a higher order modulation, namely 256 quadrature amplitude modulation (QAM). This scheme of modulation allows increasing the data rate up to 33 percent more compared with IEEE 802.11n standard which was using 64QAM [18].

However, the more revolutionary concept introduced in this standard is the support of multiple concurrent transmissions by implementing “Multi-user Multiple-Input Multiple-Output”, MU-MIMO.

By the use of smart antennas in the APs enables having MU-MIMO systems with spatial streams go until eight, meaning a speed increased of the 100 percent with respect of using just one stream. Multi-user MIMO systems present multiple advantages for the Wireless Local Area Networks, such as [19] [20] [21]:

- Efficient spectrum use
- Higher system capacity
- Decreasing latency

Moreover, MU-MIMO systems allow to do beamforming in transmission which adds more advantages to the previous mentioned, making the standard 802.11ac a valuable technology. Among these advantages are highlighted the following [18] [22]:

- Coverage improvement
- Reliability
- Data rate performance
- Signal quality improvement

The multi-user MIMO exploits the concept of spatial diverse multiplexing, thus the method of medium access used in the 802.11ac standard is “Spatial Diversity Multiple Access”. SDMA specifies that the streams are not separated by frequency or time, but instead they are resolved in space [23]. SDMA concept has been previously introduced in its predecessor, the 802.11n standard.

## 3.2 WLAN 802.11ad standard

The 802.11ad standard is the evolution of the 802.11ac standard, increasing the data throughput speeds up to 7 Gbps working at the frequency band of 60 GHz.

The use of the 60 GHz frequency band is to reduce the interference due to the levels of bandwidth. However, 802.11ad standard has some drawbacks such as the reduced range of coverage, measured in a few metres (1-10m), due to the use of millimetre frequencies.

Nevertheless, the main objective of this standard is not to provide communications with great coverage, the aim is the use of the 802.11ad standard for very short range communications with high volume data transfers such as needed for downloading files and HD video transfers. When longer ranges are needed standards such as 802.11ac can be used [24] [25].

During its standardization process, the IEEE and Wireless Gigabit Alliance worked together, thus the 802.11ad is also known as “WiGig Standard” due to marketing reasons.

Since the standard 802.11ad uses the frequency band of 60 GHz, it allows to have high speed direct communications with devices located in a short range whether there exists Line-Of-Sight between the transmitter and the receiver. Otherwise, in presence of any obstacle it could be impossible to establish the communication [24].

Similar to the previous standard, to get high connections (up to 7Gbps) it is necessary to implement smart antennas in the AP. These antennas will allow to do beamforming in transmission increasing in that way the spectral efficiency, data rate and reliability[26] .

The modulation formats used in 802.11ad are different depending the link considered. In the uplink the following modulations are used: BPSK, QPSK or 16-QAM and the way of access to the medium is “Single-Carrier Frequency Division Multiple Access”. In downlink higher order modulations are used, such as QPSK, 16-QAM or 64-QAM and the way of access to the medium is “Orthogonal Frequency Division Multiple Access” [24].

The different formats of accessing to the medium in uplink and downlink are because of mobile device battery restrictions. The use of the OFDM in mobile devices will



demand a huge amount of battery that mobile devices cannot support due to the small batteries they have implemented. Thus, an alternative method to OFDM known as “Single-Carrier FDMA” is implemented in the uplink.

The OFDM is a method of transmission that uses multiple subcarriers equally spaced and modulated with a low data rate. These subcarriers are orthogonal between them in order to avoid mutual interference. Thereby, OFDM allows to transmit large number of data but splitting them across all the subcarriers to increase the resilience against the fading due to the multipaths [27].

On the other hand, the standard 802.11ad allows to use more bandwidth even than the previous standard, 802.11ac. The channel bandwidths are 2.16 GHz, compared with 802.11ac that has maximum 160 MHz, is an important improvement.

### **3.3 WLAN 802.11af standard**

The 802.11af standard is also commonly known as “White WiFi” due to the fact the standard defines the use of Wireless Local Area Networks (WLAN) in the “white spaces” of the radio frequency (RF) spectrum [28].

This standard addresses the need from the users of having more available spectrum by providing operation in the unused TV channels in the band of Very High Frequency (VHF) and Ultra High Frequency (UHF) that comprises the range from 30-300 MHz and 300-3000 MHz, respectively. Although it is possible to use the white spaces in the whole range from 30-3000 MHz, the operating frequency range commonly used is between 470-790 MHz [29].

The utilization of unused spaces in RF spectrum has an important restriction and it is that the systems use White Wi-fi, 802.11af do not have to interfere to the primary users that have the operating frequency defined. On the other hand, this concept presents an important advantage that is the high spectral efficiency achieved with this standard, since 802.11af standard leads to use efficiently the unused frequencies.

The IEEE 802.11af allows to use channels of 6, 7 and 8 MHz of bandwidth which permits backward compatibility with the existing international TV band allocations. Its operation may be configured allowing to bond up to 4 channels, either contiguously or in two non-contiguous blocks, in order to permit the devices aggregate sufficient spectrum in a fragmented TV band spectrum and provide high data rates

[28].

The maximum data rate that can be achieved, if only just one channels is considered, is 26.7 Mbps for channels of 6 and 7 MHz of bandwidth and 35.6 Mbps for channels of 8 MHz. Meanwhile, a maximum of 426.7 Mbps can be achieved if four channels of 6 and 7 MHz are bonded, or 568.9 Mbps whether four channels of 8 MHz are bonded.

The modulation formats used in this standard range from BPSK to higher order modulation, 256QAM. The method of access to the medium is OFDMA to the downlink and SC-FDMA as the previous standards.

Following the technology implemented in the previous standard, 802.11af also allows MIMO operation using 4 streams and MU-MIMO operation by implementing smart antennas in Base Stations.

However, one of the challenges of this standard is to provide geolocation database access to unused frequencies, allowing users make use of these frequencies based on location and time of day [28]. Having knowledge of what channels are being used in the geographic part in which the user is, it is possible to avoid make use of used channels and avoid interferences.

Besides the geographic sensing, the concept of cognitive radio (CR) is also introduced as a method to avoid the interference derived by making use of spectrum spaces.

Cognitive radio is a long-term development that is becoming more of a reality because of the advances in technology. The general idea that is intended to be achieved with cognitive radio, is to define a new concept of radio that it is aware of the environment and the network. The tx-rx operations should be defined in such a way that decisions are to be made based, for example, on the level of the channels occupancy, free channels, channel quality (modulation, data rates, etc.,) [30] [31] [32].

### 3.4 WLAN 802.11ah standard

With the emerging notion of the Internet of Things (IoT), the IEEE developed the 802.11ah standard that aims to provide and specify the mechanisms that allow the connection between sensors and devices supporting the IoT requirements.

The operating frequency band used in the IEEE 802.11ah is below 1 GHz. The main reason to use this frequency is due to the large coverage the IoT applications need. Most of them require a range of coverage greater than 1km in order to interconnect devices for fixed, outdoor or point-to-multipoint applications.

Moreover, IoT applications usually operate at frequencies below of 1GHz, because of the direct relationship of the propagation losses with the frequency. These losses will be reduced significantly whether the low part of the spectrum is used.

The 802.11ah standard has defined the operating band below 1GHz excluding the use the TV white spaces (470-790 MHz), because they are just available for the previous standard 802.11af.

One of the main characteristics of the IoT applications, is that they require a large range of coverage, but low data exchange of information. Basically they exchange a little amount of data but with high periodicity compared with other services. Thus, sensors will demand, in general, low data rate tens of kbps. Thereby, the 802.11ah standard takes advantage of this characteristic and the maximum data rate can be achieved goes from 150 kbps to 346.666 Mbps, according to which order of modulation, code rate and bandwidth are used [33].

The modulations formats specified in the 802.11ah are BPSK, QPSK, 16QAM, 64QAM and 256QAM. The method of access to the medium is newly OFDM due to its great characteristics.

The 802.11ah amendment specifies narrow-band systems compared with the previous 802.11ac/ad/af standards, existing two categories of bandwidth modes.

- The first mode presents bandwidths of 1, 2 MHz. These bandwidths are mandatory for all the stations since they are globally interoperable modes optimized for sensor networking.
- The second mode defines bandwidths of 4, 8 and 16 MHz bandwidths. The use of these are optional and they will be used in those cases in which are required higher data rates (maximum 10 Mbps).

Besides the coverage range challenge, IEEE 802.11ah has to deal with other important challenges, such as [34]:

- **Supporting a large number of devices.**

IoT networks are mainly characterized by the large number of sensors that forms them. Usually, this large number of devices tries to communicate simultaneously, thus it results in multiple collisions and failed communications.

Thereby, the IEEE 802.11ah standard is intended to solve this concurrence problem and to avoid collisions, managing the communication of the large number of devices of a IoT network.

In order to do that, the IEEE 802.11ah standard defines a new method of medium access, called “Restricted Access Window (RAW)” that allows reducing collisions. The basic principle of this access method consists of classifying stations into different groups and restricting channel access only to a group of them at a particular time period [34]. Doing this achieves to improve the channel efficiency besides avoid interferences.

Thanks to this access method the number of supported devices per Access Point (AP) is 8191.

- **Low Power Consumption.**

IoT devices are battery driven, usually the battery of these devices runs out in days or weeks. Therefore, low power consumption is the challenge intended to achieve by this amendment.

IEEE 802.11ah standard incorporates methods that allow reducing the power consumption of the devices. Some of the proposed methods are the following [34]:

- Employing different wake up timers. These timers allow to wake up idle systems when they are required to make any action.
- Reducing the overhead of messages due to shorter headers and implicit acknowledgement mechanisms (ACK frames not required in some cases).
- Shortening transmission time. Implementing the function “Target Wake Time (TWT)” allows to the AP to set up the time or set of times for devices to access to the medium.

The concept of “Traffic Indication Map (TIM)” is newly re-defined in this standard. The IEEE 802.11 legacy defined a bitmap, TIM, to indicate to any sleeping station to listen if the APs have buffered frames for it. This concept is also defined in this amendment.

### 3.5 WLAN 802.11ax standard

The latest advances in technology have made WLANs has to face two main challenges [1]:

1. Address dense scenarios with a large number of Access Points (APs) and devices connected. These scenarios are intended to improve the coverage in new areas and provide high data rates.
2. Secondly, the evolution of the Internet has implicit the usage and the demand by the user of real-time and high-definition audio and video contents.

In order to deal with these new challenges, the High-Efficiency WLAN (HEW) study group developed a new amendment called IEEE 802.11ax. This amendment is considered as the successor of the 802.11ac standard due to the features included.

Mainly, IEEE 802.11ax is focused on improving two key features:

- Improvement the throughput per user in high-density scenarios (trains, stadiums, etc.,).
- Increase the battery life duration managing efficiently the power consumption of the devices.

Therefore, 802.11ax implements new mechanisms as in the physical layer (PHY) as in medium access (MAC) layer to reach these key features with high reliability in crowded environments.

- **Physical Layer improvements: high throughput.**

IEEE 802.11ax introduces new improvements in the physical layer compared with the previous standards. However, it maintains the backward compatibility with 802.11a/b/g/n and ac standards.

This new standard allows the use of two different frequency bands: 2.4 GHz and 5 GHz with the same channel bandwidths as presented in 802.11ac standard: 20, 40, 60, 80+80 and 160 MHz. As before, in those cases in which higher throughput is needed, it is possible to bond up to 8 channels.

Although the channel widths are similar to 802.11ac the data rates reached with this standard are higher due to the new modulation schemes implemented. The highest order modulation is now available is 1024-QAM.

Therefore, configuring this modulation scheme, the highest channel bandwidth and bonding up to 8 channels, it is possible to reach data rates of 10 Gbps. Whereas if only one channel is considered the maximum data rate attainable is 600 Mbps.

Moreover, the channel capacity can be also doubled if the AP and the device transmit simultaneously, implementing a “Full-Duplex” communication. However, this concept has a drawback, since it requires the implementation of any collision avoidance or collision detection mechanisms. Nevertheless, the standard pretends to solve, at least partially, this drawback by implementing medium access techniques that establish the time or set of times to access to the medium to each device.

Due to the fact that IEEE 802.11ax standard is intended to operate in ultra dense environments, new mechanisms are required to implement to improve the performance in these environments characterized by a large number of multipaths and fadings. Thereby, OFDMA is continued to be considered as the medium access technique, however some features of it has been modified to deal with the issues previously mentioned.

802.11ax amendment defines larger OFDM FFT sizes (4x larger), narrower subcarrier spacing (4X closer), and longer symbol time (4X) than the original OFDM scheme for improved robustness and performance in multipath fading environments and outdoors [1]. Table 3.2 shows the new sizes and compares with the predecessor 802.11ac standard.

**Table 3.2** *OFDM improvements IEEE 802.11ax vs 802.11ac [1]*

	<b>802.11ac</b>	<b>802.11ax</b>
FFT sizes	64, 128, 256, 512	256, 512, 1024, 2048
Subcarrier spacing	312.5 kHz	78.125 kHz
Symbol duration	3.2 us + 0.8/0.4 us CP	12.8 us + 0.8/1.6/3.2 us CP
OFDM		

By increasing these sizes in OFDM by a 4-factor allows dividing the total data information intended to transmit into more subcarriers, providing higher

robustness against multipaths and fading, although the raw link data are the same as 802.11ac.

Besides, MU-MIMO techniques are implemented in both uplink and downlink, well to deal with these issues and improve spectral efficiency.

- **Medium Access Layer improvements: high efficiency**

Another of the challenges intended to fulfil in 802.11ax is to get an efficient use of the spectrum resources. One of the mechanisms introduced to achieve this is “Dynamic Bandwidth Channel Access (DBCA)”. Implementing this concept, WLANs can adapt to the instantaneous spectrum occupancy by using the available channel width at each transmission [35]. Thereby, it achieves filling the spectrum gaps fairly.

Moreover, 802.11ax standard implements a new mechanism that improves the spectral efficiency and the system performance, based on assigning a colour code to the Basic Service Sets (BSS).

A BSS is a key component of the 802.11 networks architecture since these networks are built based on this concept. Exactly, a Basic Service Set is a set of stations (devices) that can communicate with each other and all of them are associated to one AP. These stations are connected to the medium through a Network adapter or NIC and the AP acts as a master to control the stations within that BSS [36]. Each BSS has an identifier named “Basic Service Set Identifier (BSSID)” that is the MAC address of the wireless access point (24-bit identifier) and is always associated with one BSS.

However, due to the large number of stations can be connected to the network there exists “Overlapping Basic Service Sets (OBSS)” that need to be taken into consideration. Thus, the 802.11ax standard proposes a method based on a colour code that allows APs negotiate the better channel resource allocation. Thereby, 802.11ax standard includes in the PHY preamble a 6-bits field named “BSS colour” which is the identification for each BSS. When a station receives a frame, firstly it will check the “BSS colour” or the MAC address and if the colour matches with the one its AP has announced, then the frame is considered as an intra-frame, otherwise it will be considered as an inter-frame from an OBSS.

If the frame is considered as an inter-frame, the station will ignore it since it will consider it as traffic from a neighbour BSS and that the medium is busy. In such a way that this traffic coming from neighbour BSS would not

create unnecessary channel access contention [1]. Hence, this mechanism provides a better spectral resources allocation since it avoids assigning resources inaccurately to BSS.

The BSS colour concept addressed here was already addressed in GSM in order to reuse frequencies in cellular networks. In GSM is called “Base Station Identity Code (BSID)” and it is a code to identify uniquely the APs which allows a mobile device to distinguish between different neighbouring APs [37]. The BSIC is formed by a 3-bits Network Colour Code (NCC) and a 3-bits Base Station Colour Code (BCC). The NCC identifies the network provider, therefore through these bits the mobile device will know to which APs can attach.

In addition to this, the 802.11ax standard borrows from the 802.11af the concept of the “Target Wake Time (TWT)” as a mechanism for AP to allocate resources for the stations. Through this mechanism, the AP negotiates with the station the time or the set of times to access to the medium. During the negotiation process, the station informs of the expected duration of the activity, hence the AP can control the level of contention.

Furthermore, the TWT function allows to reduce the battery consumption of the devices because the time are contented the devices are in idle state and they do not require as much power consumption as in active state.

### 3.6 WLAN 802.11ay standard

IEEE 802.11ay is an extension of the IEEE 802.11ad standard. Both standards are defined to work in the frequency band of 60 GHz. However, 802.11ay is intended to provide higher speed rates and greater coverage range than its counterpart.

The maximum transmission rate can be attainable with 802.11ay standard is 20-40 Gbps, depending on the configuration and the coverage distance is extended to 300-500 m. Compared with its equivalent, 802.11ad, that the maximum coverage allowed is 10m, the new standard will definitely allow to remove wires between High-Definition multimedia devices, PCs, peer to peer data synchronization and higher speed LAN [38].

This standard will improve some of the technologies already implemented in 802.11ad, such as:



- Multi-user MIMO systems with the possibility to add 4 streams to increase the data rate.
- SC-FDMA / OFDMA as medium access techniques.
- Channel bandwidths of 2.16 GHz, being possible to bond up to 4 channels and get a maximum bandwidth of 8.64 GHz.
- Higher order modulation. The available modulation schemes go from BPSK to 256QAM

Besides the technical specifications described above, IEEE 802.11ay allows to provide even more channel capacity thanks to bond channels and make use of MIMO features. Whether these capabilities are exploited, data rates of 200 Gbps can be achievable, according to the experts [39].

Furthermore of the advantage of connecting devices further than before, 802.11ay is considered a potential as a fixed point-to-point and point-to-multipoint backhaul technology. The reason is mainly derived of the costs associated to the deployment of these implementations with fiber optic, according to some providers.

Therefore, this new amendment can become a new alternative technology in deployments as cities networks and campus, where the range coverage demanded is perfectly fulfilled by this standard.

### 3.7 WLAN 802.11az standard

IEEE 802.11az is the next generation amendment proposed by the IEEE for future positioning systems in WLANs. The standard is still under development and it is claimed that it will be available to be available at the end of 2017. All the information about the status of IEEE 802.11az and documents already reported can be found at [8].

As it is under study, the functional requirements and technical specifications are not clearly defined yet. However, there are some key features that are already specified.

The aim in IEEE 802.11az standard is to enhance the accuracy of positioning systems in indoor environments, estimating the position of the mobile with an error

less than 0.1m [4]. The standard is intended to be implemented in ultra dense WLANs networks and to support, at least, 200 stations per AP. Therefore, getting an accuracy less than 0.1m is a key challenge that the standard must raise definitely implementing high performance estimation algorithms, such as Time of Arrival (TOA) and Angle of Arrival (AOA).

Besides, the new standard also pretends to cover new use cases related with the positioning, such as it is “Navigation in public buildings” [4]. The objective of the standard is to allow people wearing a smartphone, smart glasses, or other smart device, to navigate to a particular point of interest using an app inside of stadiums, stores, airport or museums, for example.

Therefore, the new amendment tries to provide better location accuracy and tracking of the users that are moving inside of buildings. In order to do this, the latency of the systems implementing 802.11az must be very low to allow locate/tracking univocally users that are moving. Furthermore, the APs supporting 802.11az measurements must be able to operate with high tracking refresh rate to provide location with high accuracy. Among the functional requirements of the standard it is defined that the maximum allowed latency for the systems is 500ms and the refresh rates range from 0.1 to 0.5 Hz [40].

Regarding to technical features, IEEE 802.11az shall provide access to one or more frequency bands for local area communication. These frequencies bands are already defined and they will be: 2.4, 5 and 60 GHz. The channel bandwidths that will be available in 802.11az will be the same as the predecessor 802.11ac standard: 20, 40, 80 and 160 MHz. However, there is not still information regarding to the maximum throughput can be reached or the modulations schemes available. It is thought, the standard IEEE 802.11az will follow the line defined by its predecessors, using OFDM as medium access technique, high order of modulations (up to 256-QAM) and high speed rates. Unfortunately, there will have to wait until the standard is in the market to define them.

### 3.8 Standards comparison

To summarize, Table 3.3 comprises the key features of all the standards presented above, which are the newest standards or white papers in 802.11\*\* community.

Table 3.3 WLAN standards comparison

IEEE Amend-ment	Operating Frequency Bands	Data Rate (Max)	MIMO	Spatial Streams	Channel Width	Approx. Range	Direct Positioning Support
802.11ac	5 GHz	6.93 Gbps	DL: MU-MIMO UL: MU-MIMO	8	20, 40, 80, 80+80, 160 MHz	100 m	No
802.11ad	60 GHz	7 Gbps	N/A	N/A	2.16 GHz	10 m	No
802.11af	700 MHz	568.9 Mbps	DL: MU-MIMO UL: MU-MIMO	4	6, 7 and 8 MHz	1 km	No
802.11ah	< 1 GHz	346.66 Mbps	–	4	1, 2, 4, 8 and 16 MHz	1 km	No
802.11ax	2.4 / 5 GHz	10 Gbps	DL: MU-MIMO UL: MU-MIMO	8	20, 40, 60, 80+80, 160 MHz (possible to bond)	10 m	No
802.11ay	60 GHz	40 Gbps	DL: MU-MIMO UL: MU-MIMO	8	2.16 GHz (possible to bond 4 channels, 8 GHz width)	500 m	No
802.11az	2.4 / 5 / 60 GHz	TBD	TBD	TBD	20, 40, 80, 160 MHz	TBD	Yes

## 4. POSITIONING-SUPPORTING WLANS AND NEXT GENERATION POSITIONING (NGP) SYSTEMS

In this Chapter, the different existing methods to estimate the position of the mobile in WLANs will be described, as well as the new methods will be implemented in Future Positioning Systems (FPS). Some of them have been already explained in Chapter 2 as the most commonly used algorithms in passive positioning systems, according to the accuracy they can achieve.

There are many different approaches that can be used to locating Wi-Fi devices, and they are mainly classified according to the parameter measured to estimate the position. The choice of one of these methods is determined by a concept named “Basic Service Area (BSA)” [41].

This parameter indicates the adequate coverage area of an AP. Within this area the users can move perfectly without losing the coverage, whether the channel characteristics are adequate and there is not shadow regions.

In Section 2.2 it has been already explained three positioning algorithms that can be used in active and passive systems: TOA, TDOA, AOA. Nevertheless, more methods exist and, although some of them are mainly applied in active positioning approaches, they present interesting features that must be known.

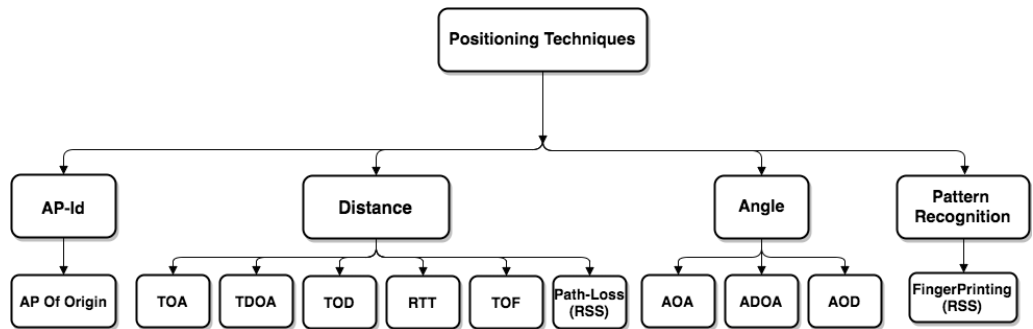
Location methods in Wi-Fi are classified into four main categories [41], based on the parameter used in the estimation:

1. **AP-Id:** estimation based on the nearest AP.
2. **Distance:** estimate the position based on lateration or trilateration with measurements of time or signal strength.

3. **Angle:** estimation of the position based on triangulation of the measurements of different angles by using directional antennas.
4. **Pattern recognition:** estimation based on the matching of a pattern, such as RSS patterns along a building, or magnetic field patterns.

Among this classification, we can highlight the following techniques that have been used in WLANs.

- AP Of Origin
- Time of Arrival (TOA)
- Time Difference of Arrival (TDOA)
- Time of Departure (TOD)
- Round Time Trip (RTT)
- Time of Flight (TOF)
- Receive Signal Strength (RSS)
- Angle of Arrival (AOA)
- Angle of Departure (AOD)
- FingerPrinting



*Figure 4.1 Wi-Fi Positioning Techniques*

Figure 4.1 illustrates the relation between these Wi-Fi techniques and the four main categories previously explained [42].

Due to the fact TOA, TDOA and AOA techniques have been already explained in Sections 2.2.1, 2.2.2 and 2.2.3, we will focus in this Chapter on the other techniques.

## 4.1 AP-Id

Methods based on cell identity are the simplest and fastest methods to estimate the position of the mobile but, also, the most imprecise. Moreover, one of the key features of this technique is that just one AP is needed to estimate the position of the mobile.

### 4.1.1 AP Of Origin

The main principle of this method consists of estimating the position by measuring the strength of the received signal. It can be implemented such as passive positioning systems as active positioning. Both approaches will be explained below:

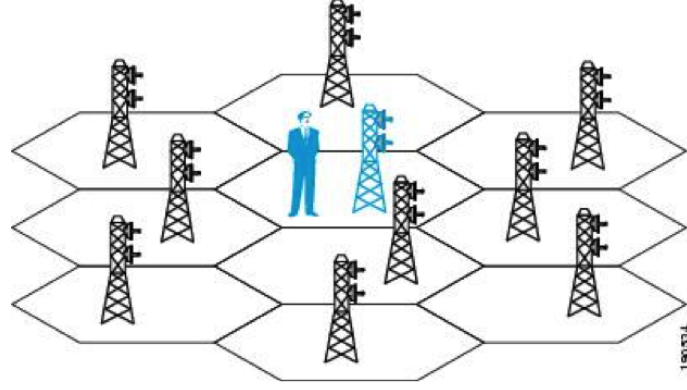
- **AP of Origin as passive positioning approach**

In this case, it is necessary to have a terminal-based topology, in which, the device is responsible of estimating its position. In this case, the own terminal will determine its location by measuring the strength of the incoming signals from the APs.

The strongest received signal will determine the closest AP. This method uses a database that contains the IDs of the APs, therefore the device will require to the database the ID of the strongest signal [41]. The server will provide the ID of the AP and its coordinates. Thereby, the position of the device is calculated by applying a predefined radio propagation model using that information. The AP position needs to be known and transmitted via some signalling to the receiver. The AP positions however are often not know or not fixed, and this can hinder tremendously the position estimation accuracy with this method.

- **AP of Origin as active positioning approach**

In network-based topologies, the APs are in charge of estimating the position of the users. In this case, the APs collect information from the mobile devices, which contains the mobile device ID and its received signal strength. The APs



**Figure 4.2** AP of Origin [43]

send this information to a server in which is stored the database with the AP locations [41]. The server will indicate the AP with the strongest signal at mobile position and, as before, the position of the mobile will be calculated applying predefined radio propagation models.

Despite the simplicity of the algorithm, in terms of estimating the position, it has some disadvantages that make it not suitable to apply in dense scenarios or scenarios that require high accuracy. Some of the most important are the following:

- Highly inaccurate method. The position of the mobile is estimated by applying predefined radio propagation models. These predefined models commonly do not have a complete knowledge about the scenario where is being applied, such as the channel characteristics, type of environment and the real movement of the mobile within the coverage area.
- The range of this method is equal to the range of the AP, shown in Table 3.3 for various standards, a very short radius of coverage, and it depends on the transmission levels of the APs. The low the transmission power, the less the range of estimation.
- It is not highly suitable for large scale positioning solutions. A database containing the positions of the APs is used, whether any changes are made about the infrastructure such as connecting or disconnecting APs, they must be updated in the database.

## 4.2 Distance-based techniques

These methods are intended to estimate geometrically the position of the device measuring the distance from each AP to the device. There are two main categories comprised in it, according to the parameter used in the measurements:

- **Methods based on time measurements.** These methods typically require to have perfectly synchronized clocks of the devices to obtain an accurate estimation. In addition, they also require the presence of Line of Sights between the Access Nodes and the receiver.
- **Methods based on the signal strength measurements.** On the contrary, these methods do not require any type of synchronization but they are susceptible to interferences and to the environment. Another advantage is that LOS presence is not required in RSS-based approaches.

### 4.2.1 Time Of Departure (TOD)

The time of departure algorithm is pretty similar to TOA, however in this method the time of departure is measured at the AP side.



*Figure 4.3 Time of Departure (TOD)*

As it is illustrated in Figure 4.3, is the equal to TOA in concepts but changing the origin of the signal. Therefore, it will be characterized by the same features than as TOA, which are summarized in the Table 4.1 as advantages and disadvantages.

### 4.2.2 Round Time Trip (RTT)

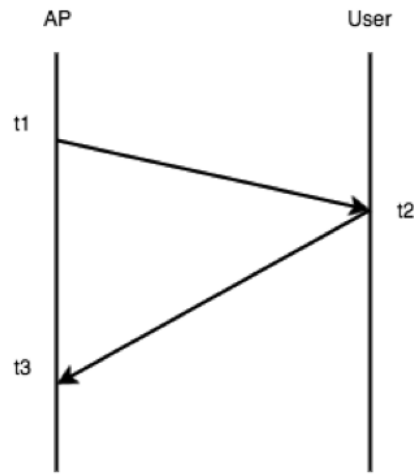
This method estimates the distance from the AP to the devices by measuring the time it takes the signal from an AP to a mobile device and back again. In contrast



**Table 4.1** *TOD features*

Advantages	Disadvantages
High accuracy	Requires perfect synchronization between the clocks.
Low cost	Affected by noise environments
Not require extra HW	—

with TOA, TOD or TDOA, with this method it is necessary to measure the time in the 2-way of communication, as it is represented in Figure 4.4.

**Figure 4.4** *Round Time Trip (RTT)*

Looking at Figure 4.4, the total round trip time can be easily calculated by applying the equation 4.1:

$$RTT = t_3 - t_1 \quad (4.1)$$

Where:

$t_3$  = time of arrival of the incoming signal from the mobile

$t_1$  = time of departure of the signal from the AP

The distance between the AP and the device, is then determined by equation 2.4:

$$d = c \cdot RTT/2 \quad (4.2)$$

Unlike TOA and TOD, the RTT does not require perfect clock synchronization. However, RTT measurement has to be reported to the location server where the distance from the AP to the device is calculated [44].

### 4.2.3 Time Of Flight (TOF)

The Time of Flight measurement becomes determined by equation 4.3, and the distance is estimated applying the relation 4.4:

$$TOF = \frac{(t_4 - t_1) - (t_3 - t_2)}{2} \quad (4.3)$$

Where:

$t_1$  = time of departure of the signal from the AP

$t_2$  = time of arrival of the incoming signal from the AP

$t_3$  = time of departure of the signal from the mobile

$t_4$  = time of arrival of the incoming signal from the mobile

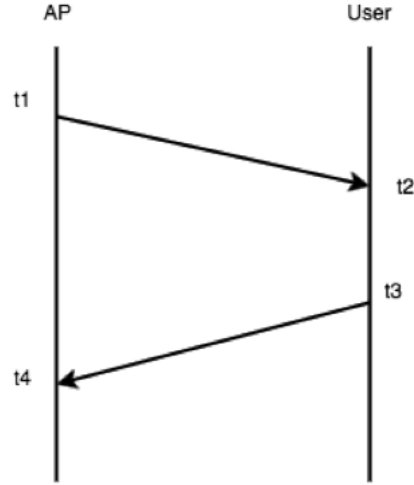
$t_3 - t_2$  = processing time

$t_4 - t_1$  = RTT

$$d = c \cdot RTT \quad (4.4)$$

This method is considered as a variant of the previous one, RTT. One of the disadvantages of RTT is that it does not consider the processing time that device needs to commute between reception and transmission.

Therefore, TOF takes into account the processing time at the device side when it is estimating the time of flight of the signal. Thereby, the distance is then estimated by measuring the RTT and subtracting it the processing time in the device. Figure 4.5 illustrates the working principle of this method.



**Figure 4.5** Time Of Flight (TOF)

#### 4.2.4 Received Signal Strength (RSS)

RSS technique represents another different approach based on lateration to compute the distance between an AP and the device. The objective consists of measuring the received signal strength on the device. One of the characteristics of this approach is that it can be used either by the AP as the devices, therefore, it can be considered equally in active and passive systems.

This method uses the propagation losses in WLAN to calculate the distance between the AP and the device. As we know, the propagation losses are closely related with the distance between transmitter and receiver, as indicated in 4.5.

$$P_{RX} = P_{TX} - 10n \cdot \log_{10}(d) + \eta \quad (4.5)$$

where:

$\eta$  = is the measurement of noise + shadowing

The parameter “n” describes the *path loss exponent* and it indicates how far the propagation losses increase as the distance does. Its value depends on, mainly, the environment, being usually tabulated for some well-known environments, such as free space whose value is 2.

The path loss model in eq (4.5) is the most used one, but finding a suitable and

accurate path loss model is not a trivial task. In multi-floor indoor scenarios we have to consider for example also the floor and wall losses in the path loss model, while for outdoor propagation the type of terrain (urban, sub-urban,...) can also influence the path loss modeling. Finding the suitable path loss model is one of the critical points in using the path-loss-based positioning estimate.

However, the propagation losses do not consider other losses that affect to the received signals, such as cable losses. Thus, in order to obtain accuracy results with this method, these extra losses should be known and added to the propagation losses to obtain the total losses of the medium.

As it has been mentioned, RSS algorithm determines the distance between transmitter and receiver measuring the received signal strength. Therefore, given the transmission power, the gains and the total losses known, the distance can be easily derived as it is given in 4.6.

$$d = 10^{((P_{TX} - P_{RX} + \eta)/10n)} \quad (4.6)$$

Thereby, the calculated distance allows plotting a circle around the AP in which the target is located. As in time based measurements, to estimate the position of the user univocally, three Receive Signal Strength measurements (RSS) are needed.

The value of the RSS can be obtained of two different forms, by the network or the user:

- **By the network:** the network reports the RSS values at which it receives mobile device transmissions [43]
- **By the user:** the device reports the values of the RSS of the received network transmissions.

This method presents an important drawback and it is the periodicity with which the RSS measurements are reported. This periodicity depends on the radio vendors, thus it cannot be controlled by the algorithm to enhance the accuracy. Furthermore, RSS method accuracy is significantly affected by the propagation anomalies of the environment. Thereby, its suitability will be determined by the type of environment in which apply.

However, it presents an interesting advantage, due to the fact it does not require implementing any specialized hardware to achieve the estimation and any synchronization between devices.

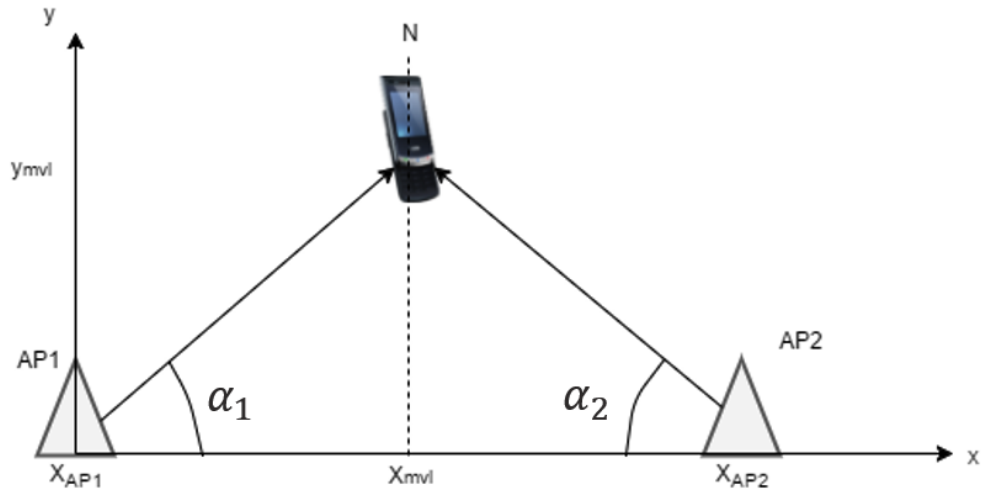
### 4.3 Angle based techniques

The arrival of MIMO systems have made possible to develop new positioning techniques such as, angle based techniques. These techniques are based on measuring different angles at the antenna arrays in the receiver/transmitter and apply triangulation to estimate the position of the mobile.

#### 4.3.1 Angle Of Departure (AOD)

AOD method exploits the same concept as AOA, however the angle measured is different. In AOA, as it is explained in Section 2.2.3, the position of the mobile is estimated by measuring at the receiver side, the angle of arrival.

Nevertheless, with AOD approach, the estimation of the position of the mobile consists of measuring at the transmitter side, the angle of departure from the AP. Figure 4.6 presents this principle.



*Figure 4.6 Angle Of Departure (AOD) principle*

In this case, the position of the mobile in 2D will be determined by the equations 4.7 and 4.8.

$$\alpha_1 = \operatorname{atan2} \left| \frac{y_{mvl} - y_{AP1}}{x_{mvl} - x_{AP1}} \right| \quad (4.7)$$

$$\alpha_2 = \operatorname{atan2} \left| \frac{y_{mvl} - y_{AP2}}{x_{mvl} - x_{AP2}} \right| \quad (4.8)$$

Equations 4.7 and 4.8 are quite similar to the equations obtained for AOA estimation 2.3 and 2.4, the difference stays in the way the angle is now measured in AoD approaches.

### 4.3.2 Angle Difference Of Arrival (ADOA)

This method is similar to TDOA approach but measuring the angle difference and using it to estimate the position of the mobile. With ADOA minimum two AP are always required to estimate the position of the mobile.

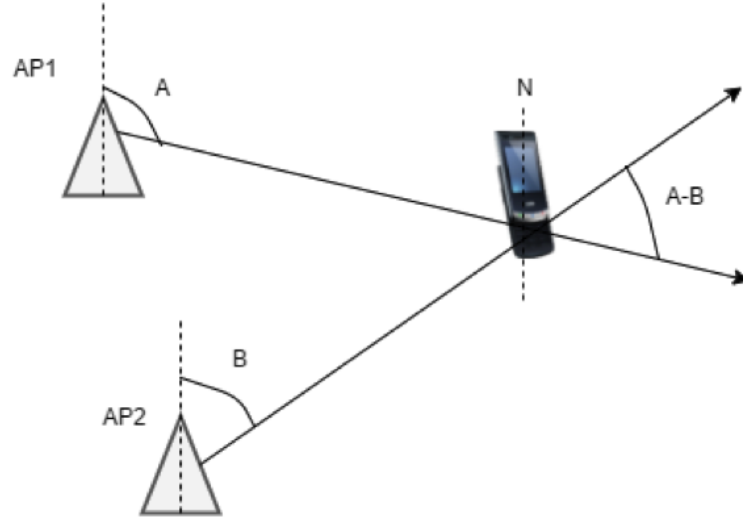
ADOA algorithm is an algorithm that can be used as well in passive positioning systems because the position of the user is calculated by the own device. The ADOA method calculates the difference of the two arrival angles at the receiver side. With these measurements the mobile is able to estimate its position. In Figure 4.7 is illustrated this principle.

As in the other algorithms of this family, ADOA accuracy is significantly dependent on the accuracy of the array antennas used at the receiver and the directionality of the array antennas used at the transmitter. Moreover, this algorithm generates a special cost to the networks in which is implemented, due to the requirement of installing smart antennas.

Nevertheless, it does not require any sort of clock synchronization between devices, which is an optimal solution to those scenarios where the Time based positioning techniques cannot be applied.

## 4.4 Pattern Recognition

Unlike the techniques explained so far, pattern recognition techniques are based more on evaluating the matching between the measurements and the reference values. The



**Figure 4.7** Angle Difference of Arrival (ADOA)

most known technique based on this approach which is used in WLAN environments to positioning is called **fingerprinting**.

Fingerprinting is also known as “*Location Patterning*” algorithms [45] [46]. This is because they make use of a database containing the signal patterns, which has to match with the signal measurements for positioning.

The estimation of the position using Fingerprinting approach is divided into two phases:

- I. **Phase 1**, called *Training-phase*. In this phase, the area in which the algorithm will run is covered with a pattern of recording points, called fingerprints [41]. For each fingerprint a measurement of interest such as RSS or magnetic field must be collected. The result will be a vector  $R$  for each fingerprint with each characterizing a certain parameter value per each AP (e.g., its RSS).
- II. **Phase 2**. At this phase, all the collected measurements are stored in the database that will be used in the algorithm. The database is called *radio map*.
- III. **Phase 3, Estimation phase**. This is where we make the estimation based on pattern matching with the database stored at phase 2.

When these two phase have been executed, Fingerprinting method can be launched

and make use of the database of the environment to determine the position of the mobile.

Firstly, RSS measurements of the all APs in the environment have to be collected. These measurements will be stored in other vector,  $P$ , which will be compared with  $R$  to get the user position. The algorithm calculates the matching between the vectors  $P$  and  $R$  by applying metrics such as: euclidean distance, gaussian likelihood, rank-based approaches, Mahalanobis distance, etc., [41] [46] on the radio map, and the position of the mobile will be in the area with smallest unmatched.

The main drawback of this method is that it needs to carry out some offline steps before the algorithm is ready to use, such as the collection of the RSS measurements and the creation of the database. Furthermore, it brings other drawback that is the lack of scalability, because of the number of APs comprising the scenario must be known in phase 1 and they can change their position or characteristics only in this phase, unless this phase will be repeated before executing the algorithm.

Nevertheless, fingerprinting positioning is a very accurate algorithm and it allows solving some of drawbacks of the previous methods:

- It does not need any hardware modification such as Angle based techniques required.
- No clock synchronization is required to estimate accurately the position.

## 4.5 Next Generation Positioning Systems (NGP)

IEEE 802.11az is the amendment that specifies the requirements for the Future Positioning Systems (FPS), also referred to as NGP. As it has been described in Section 3.7, these NGP will require accuracy of 0.1m and they are intended to support 200 devices per AP, working at 2.4, 5 and 60 GHz.

The proposed positioning algorithms in this new generation systems to achieve these characteristics are mainly a modification of TOA, based on time of flights, and AOA. Although, AOA algorithm can be used as it has been defined in Section 2.2.3, TOA algorithm is slightly modified to have into consideration some new parameters introduced in 802.11az standard.



The 802.11az amendment proposes a network comprises by large number of APs in order to cover ultra dense scenarios. All of these APs will be sending periodically information to the network in order to help the mobile to locate itself. The information sent from the APs is used by the user to determinate, in combination with TOA algorithm, his/her location as it is illustrated in Figure 1.1.

Therefore, the TOA-based algorithm proposed in NGP is a non-linear problem defined by the equation 4.9.

$$t_i = t_s + \frac{d_{mvl,AP_i}}{c} + \sum_{j=2}^i \left( \frac{d_{AP_j,AP_{j-1}}}{c} + t_f \right) \quad (4.9)$$

Where:

$t_s$  = time of start,reference of time measured respect  $AP_1$

$t_f$  = time forward distance between APs in time units

$d_{mvl,AP_i}$  = distance between mvl and  $AP_i$

$d_{AP_j,AP_{j-1}}$  = distance between  $AP_j$  and  $AP_{j-1}$

$t_i$  = estimated time of arrival from  $AP_i$

The parameter  $t_f$  is specified as a constant value and equal to all the APs since it is derived by the network architecture. Its value is not yet specified in the reports that IEEE has already published [8].

The proposed method has several advantages over the traditional positioning approaches. Mainly it reduces the overhead generated in the network by other methods, where the required messages exchange to positioning the user overloads the network. With this approach, the APs are sending frames through the network but the user is not sending any frame back to the APs, reducing in this way the overhead.

However, this method increases the complexity in the location estimation process, due to its non-linearity, although thanks to this complexity in the estimation process, the proposed method allows to get accurate position (0.1 m in accuracy).

## 5. SIMULATION-BASED STUDIES

In order to study the proposed passive positioning algorithm for the IEEE 802.11az standard, a simulator in Matlab has been built. Thereby, along this chapter we will make an introduction about the basis on which the simulator model has been built, explaining as well the simulator and its functionalities. Finally, we will present the results obtained for the study cases accomplished, in order to get some conclusions and evaluate the features of the algorithm implemented.

### 5.1 Simulator modelling

#### 5.1.1 Introduction to the model

As it can be seen in equation 4.9, the proposed algorithm in NGP results in a nonlinear problem solving. Therefore, a nonlinear method must be implemented to solve the problem and find a solution of the mobile position.

The passive positioning algorithm addressed in this thesis work estimates the position of the mobile based on the TOA and AOA measurements, applying a recursive non-linear method and following the next methodology:

1. Supposing we know the position of the APs and the mobile in the network, we compute the time and angle observables, applying the equations 4.9, 2.3 and 5.3:  $t, \alpha, \theta$ .
2. Assuming an initial guess point,  $x_0$  (x,y,z) and applying the equations , we compute the time and angle estimated observables:  $t_{est}, \alpha_{est}, \theta_{est}$ , which are affected by noise effects.

$$t_{est_i} = t_s + \frac{d_{mul,AP_i}}{c} + \sum_{j=2}^i \left( \frac{d_{AP_j,AP_{j-1}}}{c} + t_f + n \right) \quad (5.1)$$

$$\alpha_{est_i} = atan2 \left| \frac{x_{mvl} - x_{AP_i}}{y_{mvl} - y_{AP_i}} \right| + n \quad (5.2)$$

$$\theta_{est_i} = asin \left| \frac{z_{mvl} - z_{AP1}}{d_{mvl,AP1}} \right| + n \quad (5.3)$$

where  $n$  represents the noise.

3. With the estimators obtained we create the objective functions for each positioning technique. The objective function for TOA and AOA methods are defined by equations 5.4 and 5.5, respectively:

$$f_{TOA} = \begin{bmatrix} t_{est1} \\ t_{est2} \\ \dots \\ t_{est_i} \end{bmatrix} \quad (5.4)$$

$$f_{AOA} = \begin{bmatrix} \alpha_{est1} \\ \alpha_{est2} \\ \dots \\ \alpha_{est_i} \\ \theta_{est1} \\ \theta_{est2} \\ \dots \\ \theta_{est_i} \end{bmatrix} \quad (5.5)$$

4. By the recursive approach we will minimize the objective function. From the initial guess point  $x_0$  it will try to minimize the objective functions  $(t, \alpha, \theta)$  in order to get the closest solution of the estimated position to the real position of the mobile.
5. The value of  $x, y, z$  which gets the minimum value of the function will be the estimated position of the mobile:  $x_{est}$

### 5.1.2 Non-linear methods

There are a wide variety of nonlinear methods that can be implemented to find a solution for nonlinear systems. However, not all of these methods suit in the thesis

approach due to the casuistry of the thesis and the limitations of these mentioned methods.

These nonlinear methods are commonly classified according to parameters such as: convergence, accuracy, computational cost, stability, easy-implementation, etc., Thus, the method that suits better in the thesis approach is a method which presents a great grade of convergence, accuracy and stability; since the aim of this thesis work is to develop a high accuracy algorithm able to add new APs to emulate dense networks. However, among all the available non-linear methods we will focus in this thesis on: Newton-Raphson and Non-linear squares methods.

### Newton-Raphson method

The Newton-Raphson method finds iteratively better approximations to the roots (or zeroes) of a real-valued function [47].

$$x : f(x) = 0$$

One of the characteristics of Newton-Raphson method (N-R) is its fast rate of convergence and how it enhances the grade of convergence in each iteration. These characteristics make N-R method the one that a priori fulfils our requirements.

N-R method solution is based on the Taylor series equation 5.6, due to the fact it finds the solution by approximating to root of the equation derived, as it is given in equations 5.7 and 5.8.

$$f(x) = f(x_0) + f'(x_0)(x - x_0) + \frac{f''(x_0)}{2!}(x - x_0)^2 \quad (5.6)$$

$$f(x) = 0 \approx f(x_0) + f'(x_0)(x_1 - x_0) \quad (5.7)$$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} \quad (5.8)$$

The equations 5.7 and 5.8 are referred to a single variable function. Nevertheless,

this method can be easily generalized and derived to a multi-variable approach as it is given by equations 5.9, 5.10 and [48].

$$x = \begin{bmatrix} x_1 \\ x_2 \\ \dots \\ x_n \end{bmatrix}, f(x) = \begin{bmatrix} f_1(x) \\ f_2(x) \\ \dots \\ f_n(x) \end{bmatrix} \quad (5.9)$$

$$J = \frac{\partial(f_1, f_2, \dots, f_n)}{\partial(x_1, x_2, \dots, x_n)} = \begin{bmatrix} \partial f_i \\ \partial x_j \end{bmatrix}_{n \times n} \quad (5.10)$$

$$x_{n+1} = x_n - J^{-1} \cdot f(x_n) \quad (5.11)$$

As it can be seen, this method uses the inverse of the Jacobian matrix to calculate the next guess point,  $x_{n+1}$ , until the algorithm converges to the true position of the MS. Thereby, since the inverse of Jacobian matrix is used, N-R method forces to the Jacobian matrix to be square, otherwise this method cannot be applied.

Square matrix restriction is an important drawback of the N-R method that affects mainly to those cases where the number of equations to be considered, " $m$ ", is greater than the unknowns, " $n$ ", ( $m > n$ ). In the case of the thesis presented here, the resulting Jacobian matrix will be non-square in almost all cases, since its size is determined by the number of the APs considered such as:  $N^{\circ}AP \times 3$ . Newton-Raphson could be applied when  $N_{AP} = 3$ , in other cases the Jacobian will be singular and N-R cannot calculate the estimated position because of the singularity of the Jacobian matrix.

However, there exist solutions that avoid the problem of singularity by modifying slightly the N-R method. One of the solutions is to consider the pseudo-inverse of the Jacobian matrix instead of the inverse. Pseudo-inverse approach allows to calculate the "inverse" of a non-square matrix and then, apply the N-R method. Thereby, the modified N-R method is given by the equation 5.12.

$$x_{n+1} = x_n - ((J^T \cdot J)^{-1} \cdot J^T) \cdot f(x_n) \quad (5.12)$$

Testing the modified N-R method, we have noticed that there are some cases in which even using the pseudo-inverse, N-R cannot estimate the position of the mobile, due to the determinant of the matrix tends to zero. This is because the Jacobian matrix contains high values (order of  $10^{11}$ ) and when the pseudo-inverse is calculated it goes to zero.

In such a way, N-R method becomes again disposable, besides singularity problem was solved with pseudo-inverse approach. The mentioned drawbacks make to discard this solution and research another approaches, such as: Non-linear squares method.

### Non-linear squares method

In order to solve the singularity problem of the N-R method, we considered the non-linear squares approach as a possible solution, due to its application for over-determined systems.

Non-linear squares method is a least squares method that finds a solution for a nonlinear system in which the number of equations is greater than the unknowns ( $m > n$ ). The principle of the method consists of approximating the non-linear system to a linear one and applies the “least squares” standard, which consists of minimizing the sum of the squares of the residuals of every equation.

Thereby, we implemented this model in Matlab using an available function of the toolbox of Matlab called “*lsqcurvefit*”. This function finds the position of the user by minimizing the equation 5.13. The basic principle of this method consists of iteratively reducing the sum of the squares of the errors between the objective function and the measured data using an input sequence, *xdata* [49].

$$\min_x \|F(x, xdata) - ydata\|_2^2 = \min_x \sum_i (F(x, xdata_i) - ydata_i)^2 \quad (5.13)$$

where [50]:

*xdata* = input data, namely the positions of the APs

*ydata* = are the observed output, namely the real time or angle observables

$F(x, xdata)$  is a matrix-valued of the same size as ydata.

$F(x, xdata)$  will be the function of the  $t$ ,  $\alpha$ ,  $\theta$  observables, respectively

The `lsqcurvefit` allows to set-up some parameters and customize the optimization process according to the use case, such as:

- Algorithm: it allows to select the algorithm used in the minimization process.
- Maximum number of iterations allowed in the minimization process.
- Lower and upper bound: bounds in which the found solution has to be.
- Tolerance: constant value that will be compared with the step of the next guess point. It is a stop condition, the algorithm will stop whether the step of the next point is less than the tolerance.

Among the algorithms available, `lsqcurvefit` method includes “*Levenberg-Marquardt*” algorithm, commonly known as LMA. This algorithm is a curve-fitting algorithm that is a combination of two methods:

- **Gradient descent method:** minimizes the objective function based on the steepest-descent direction. It starts with an initial guess point and calculates the gradient; the next point is calculated in the negative direction of the gradient, as it is given in 5.14. The process is repeated until the algorithm converges, i.e. when the gradient is zero [51].
- **Gauss-Newton:** this approach finds the local minimum by minimizing the sum of square function values.

$$x_{n+1} = x_n - \lambda \cdot \nabla f(x_n) \quad (5.14)$$

where:

$\lambda$  is a small value that forces to the algorithm to make small steps

LMA method is usually used in those cases where the N-R method cannot be implemented, as it is the case of this thesis. Therefore, we will set-up the algorithm LMA to be used in the optimization process due to its similarity with N-R method.

Nevertheless, lsqcurvefit method presents an important drawback. This method is quite unstable because it does not easily converge. The algorithm will try to find a solution minimizing the objective function and decreasing the initial guess point until the tolerance value is reached. Therefore, when this situation is achieved, the algorithm will stop and whether the found value is a minimum, the solution has been found, otherwise it will be displayed “local minimum possible”. In Figure ?? is illustrated the non-convergence problem of lsqcurvefit method for an example of the estimation error as a function of the elevation variance for TOA and AOA methods. The estimation error with AOA method should be close to 0 when the elevation variance is 0, due to the fact the other variables are put to 0. However, the error obtained is quite higher, roughly 30 m.

Those cases in which the local minimum is possible but not found, the algorithm should try with other initial guess point which could obtain the local minimum in the successive iterations, but it is not able to change the  $x_0$  in those situations.

We studied a solution to this problem by creating a loop to those cases of non-converge was reached, in which other  $x_0$  point was generated. Nevertheless, it still continued moving away of the optimal point because it is not possible to generate the correct initial guess point that will make the algorithm converges.

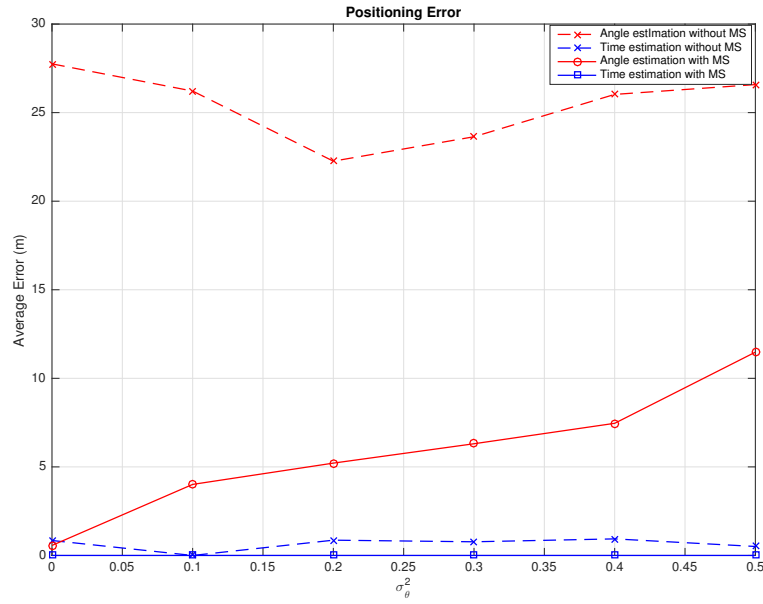
In such a way, we needed to implement an algorithm that was able to generate different starting points for those cases where the non-convergence was reached. Therefore, taking advantage of the Matlab help [52], we implemented the algorithm “*MultiStart*” which generates different starting points and uses a local solver to solve the optimization problem.

Thus, the *MultiStart* method allows defining the optimization problem to solve and the number of start points, “ $k$ ”, to run. The method will generate  $k-1$  start points using a random function and also the  $x_0$  point from the problem structure. For each one of these values, the specified optimization problem will be runned and the point in where the lowest function is found will be the solution of the optimization problem [52].



Therefore, applying lsqcurvefit method in combination with MultiStart technique we achieve to solve the nonlinear problem, getting that it converges to the optimal solution of the estimated position of the mobile.

Figure 5.1 illustrates the resulting estimation error when both methods are applied jointly in comparison with the estimation error obtained when only lsqcurvefit is considered. It can be noticed that using multistart method combined with lsqcurvefit, the AOA estimation error approaches to the expected results, getting an error much closer to 0 for the elevation variance 0 than using only lsqcurvefit algorithm.



**Figure 5.1** Estimation error using lsqcurvefit and MultiStart methods

### 5.1.3 Simulator

Based on the mentioned in previous sections, a simulator with a Graphical User Interface (GUI) has been built in order to test the functionalities of the passive positioning algorithm proposed for the IEEE 802.11az standard.

The study cases that will be carried out are based on studying the influence of the noise in the time, azimuth and elevation estimators, in order to get a good estimation of the position of the mobile. The equation 5.15 represents the general approach to study. The simulations accomplished will be focus on characterizing

the allowed values of the noise under Line Of Sight (LoS) and Non-Line Of Sight (NLoS) environments, in order to get accuracies less than 0.1, 1 and 5 m.

$$Y = Ax + N \quad (5.15)$$

Where:

$Y$  = matrix that contains the observables (time, azimuth or elevation)

$A$  = matrix with the values resulting of the descomposition of the objective function

$x$  = vector with the coordinates (x,y,z)

$n$  = noise

The influence of the environment will be modelled through the mean value of the noise. A Gaussian noise is defined by the mean and variance values as it is given in equation 5.16. The variance value indicates the power of the noise, whereas the mean value represents a measurement of how the signal is affected until it is received in the receptor, i.e. multipaths, fadings, etc. Therefore, varying the mean value of the noise we will simulate NLoS environments.

$$N = (\mu, \sigma^2) \quad (5.16)$$

Thereby, the simulator built includes a user interface to make easier its use and the configuration of all the parameters. The user interface is illustrated in Figure 5.2, and it is divided into two main parts:

1. **Parameters:** in this section it can be introduced all the input parameters to configure the study case considered.
  - **Study case:** there are 5 studies cases possible to simulate comprised into 2 main categories:
    - **Influence of two observables jointly in the estimation:** this case allows to have a global idea about how two observables affects simultaneously to the estimation. Choosing among the influence of

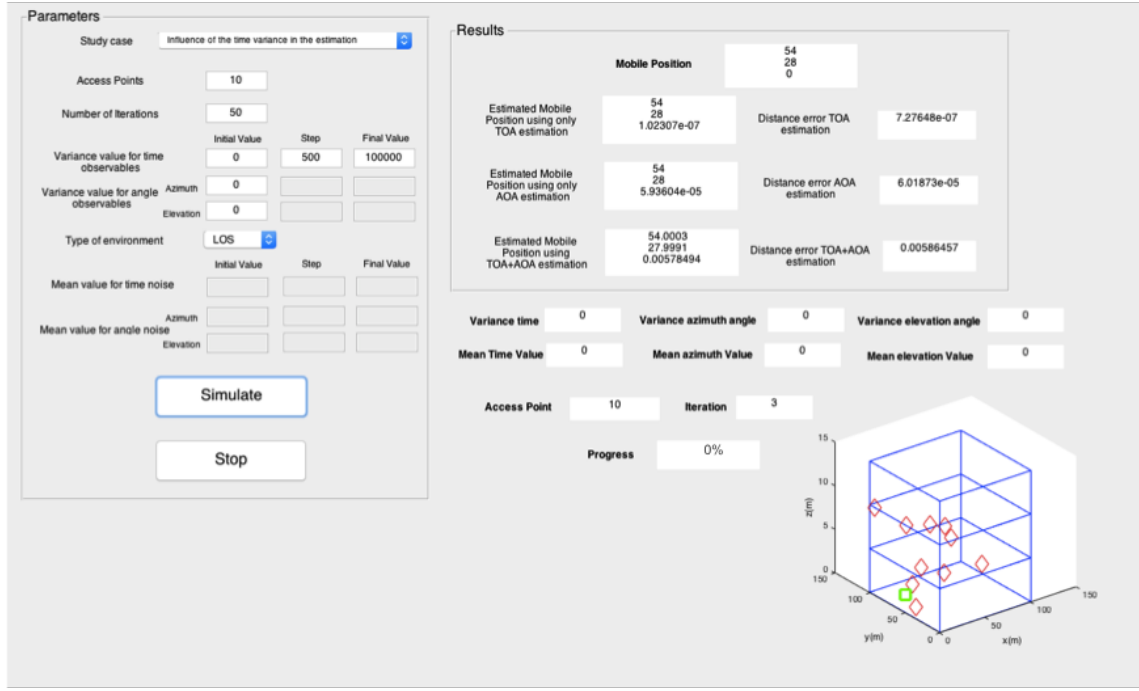
azimuth and elevation variance or the time and azimuth/elevation variance, we will obtain a mesh illustrating the variation of the distance error as a function of the range of the variances considered.

- **Influence of the observables individually in the estimation:** through this case we are allowed to study the influence of the time, azimuth or elevation variance in detail when the other variables are constant. The results will be a 2D graph illustrating the distance error versus the variance.

- **Access Points:** number of access points to place in the scenario. The simulated scenario is a building of size 100x100x15 m divided in 3 floors in which the APs and the mobile are placed randomly during each iteration. Each floor has a height of 5 m.
- **Number of iterations:** specifies the iterations that the algorithm will run. In each iteration the APs and the mobile will have a new location.
- **Variance value for time and angle observables:** defines the range of the time and angles variances to study.
- **Type of environment:** LoS or NLoS. The distinction between LoS and NLoS cases is done inside the algorithms, based on zero or non-zero mean of the observables (zero mean for LoS and non-zero mean for NLoS cases).
- **Mean value for the time and angle noise:** if the NLoS environment is selected, these boxes will be active, allowing to introduce the range of the mean values to study.

2. **Results:** is the output section, it shows the mobile position, the estimated position and the distance error with three different estimation methods: TOA, TOA and TOA+AOA. The TOA approach refers to the time observables as defined in the 802.11az standard. The AOA approach is a classical 3D AOA approach, based on the observed azimuth and elevation angles of arrival at the mobile. Moreover it shows the variances and mean values that are being simulated, such as the iteration and the progress of the simulation.

Furthermore, in the user interface it is illustrated how it is the location of the APs and the mobile in each iteration inside of the building. The red diamonds represent the APs positions and the green square represents the mobile.



*Figure 5.2 Created Matlab GUI*

## 5.2 Results

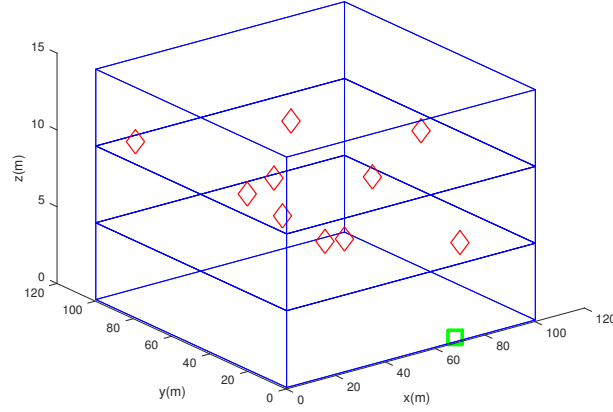
The studies carried out in order to evaluate the performance of the NGP systems have consisted basically of the following:

1. Study of the influence of the noise in the accuracy of the estimated position.
2. Study of the influence NLoS environments in the mobile position estimation.
3. Influence of the number of APs in the accuracy of the estimation process.

The methods used to determine the position of the mobile have been mainly, the modified TOA based on time of flights measurements and AOA. Nevertheless, a joint time and angle estimation has been also included, in order to know whether a joint estimation would enhance the accuracy of the estimated position and decrease the estimation error.

The study cases addressed in the following points have been done assuming we have a typical building of size 100x100x15 m, divided in 3 floors. In this building, the APs

and the mobile will be placed randomly in each iteration, the number of iterations considered in order to have a enough grade of convergence has been 50. In Figure 5.3 the used scenario is illustrated.



**Figure 5.3** *Simulated scenario*

For the two first study cases, the number of APs will be fixed to a value of 10 in order to evaluate the influence of noise and environment in a dense scenario. However, the third study is focused on number of APs impact. Therefore, the number of APs considered will be variable inside of a range from 0 to a high value, for example 100.

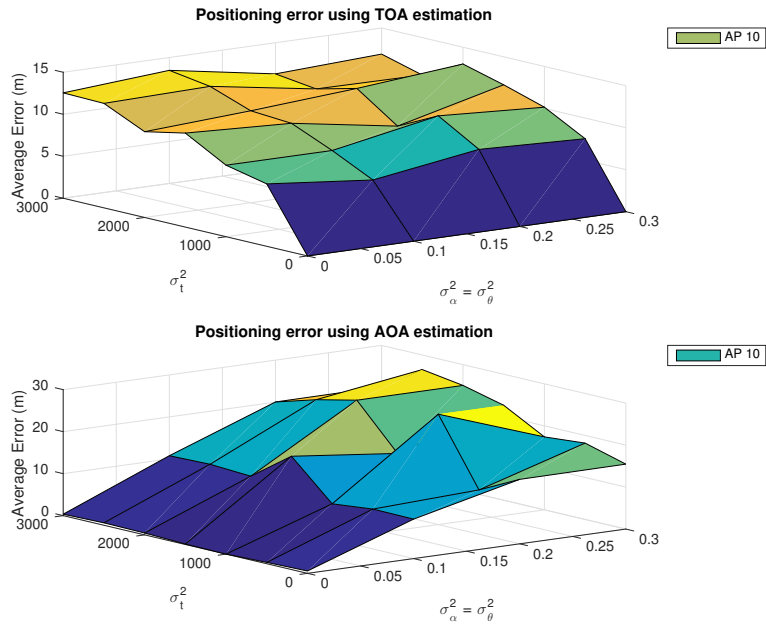
### 5.2.1 Noise influence in the estimation process

All the positioning techniques have to deal with the adverse effects of the noise, which affect to the accuracy of the estimation and also to the range of coverage. Besides this, indoor positioning techniques are limited as well by their short range of coverage. Therefore, characterizing the effects of the noise over the estimation process becomes a key issue that must be done in all the positioning techniques, but specially in indoor positioning.

Thereby, we will study in this part how the noise affects to the estimation in LoS environments, determining the maximum allowed values of noise in order to get accuracies of 1 and 5 m in the estimated position, and also for the key challenge of getting accuracies less than 0.1 m, specified in the 802.1az standard addressed here [4].

In the first place, we will obtain a general idea about how is the influence of the noise over the time and angle estimators and also how it affects to the estimation error. In order to do that, we have represented the shape of distance error as a function of the noise variation in the time and angle observables, for the two considered methods of positioning: TOA, AOA.

Assuming the azimuth and elevation variances are equal, we have varied the variance from the time observables,  $\sigma_t^2$ , initially from 0 to 3000 ns, obtaining the results plotted in Figure 5.4. In this figure is represented jointly the influence of the time and angle variances for each positioning technique, TOA and AOA. As it can be seen, the values of the angle variance,  $\sigma_\alpha^2$  and  $\sigma_\theta^2$ , do not affect in the TOA estimation, such as the values of the time variance do not in the AOA estimation. This is because both estimation methods are completely independent and uncorrelated. Therefore, the error will only vary in one direction; time or angle, depending on the estimation method is being considered, being constant in the other direction.



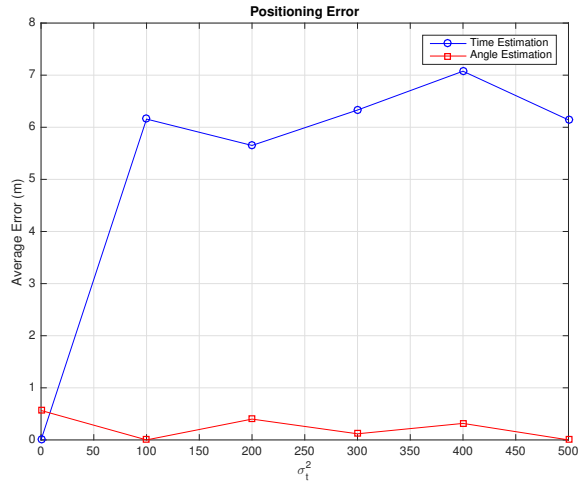
**Figure 5.4** Influence of time and angle variances in LoS environments

In addition, the error obtained with the TOA method is less than the obtained with the AOA method. This indicates the modified TOA method is more accurate than the traditional AOA implemented in this approach.

Nevertheless, the error obtained with both methods (TOA and AOA) and also for the considered range is much higher than the objective of 5, 1 and 0.1 m. Therefore, in order to get the objective accuracies the variance ranges considered in the simulation must be shortening. From the first plot of the Figure 5.4, we can see that the maximum allowed time variance for getting an position error less than 5 m is roughly 500 ns. However, we cannot do the same for the angle variance due to the fact the angle estimation comprises the estimation of two angles, azimuth and elevation. Thereby, the study of the maximum noise allowed in AOA method in order to get accuracies less than 5, 1 and 0.1m has to be done separately for each angle, azimuth and elevation.

### The influence of the time variance $\sigma_t^2$ on the positioning error

Focusing firstly on the time estimation characterization, we obtain that reducing the range of the time variance up to 500 ns the distance error represented in Figure 5.5.



**Figure 5.5** Influence of time variance in LoS environments

As it can be seen in Figure 5.5 the errors below 5 m are achievable if the variance of the time observables range from 0 to 80 ns. Thus, getting a better accuracy in this range we obtain the maximum allowed values of the time variance in order to achieve the accuracies specified, which are represented in Table 5.1.

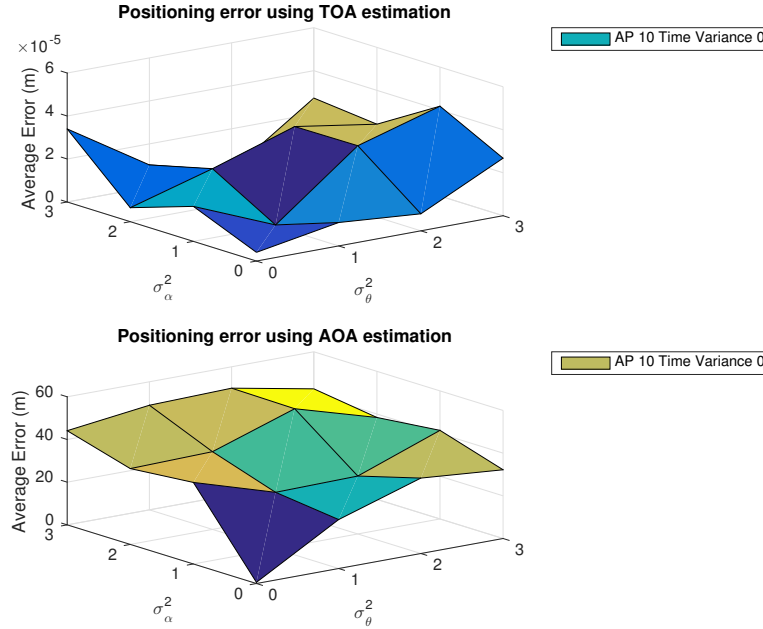
**Table 5.1** Maximum allowed time variances in LoS environments

Target maximum positioning error	Time Variance $\sigma_t^2$
5 m	80 ns
1 m	3 ns
0.1 m	0.02 ns

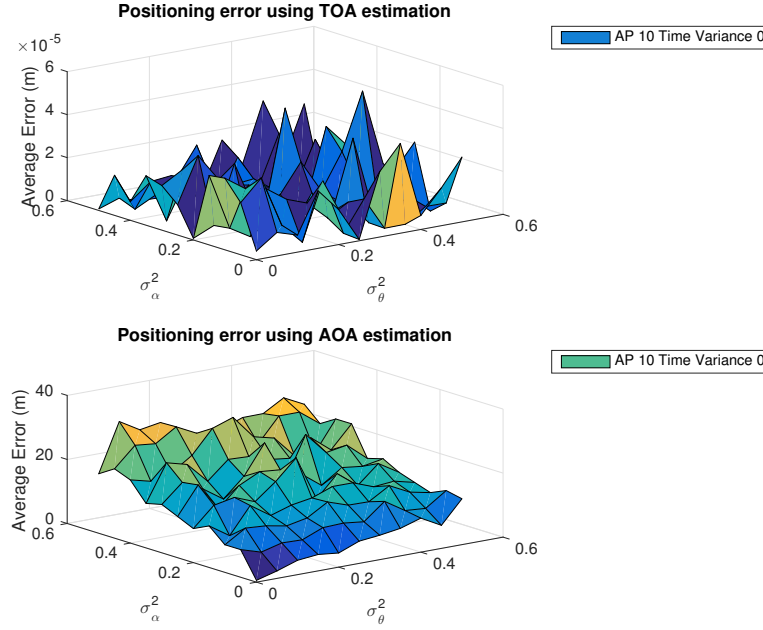
### The influence of azimuth and elevation angle variances on the positioning error

Assuming the time variance is zero, i.e. there is not any noise affecting to the time estimators, we have also studied the influence of the azimuth and elevation variance simultaneously in the AOA method.

The values of azimuth and elevation variances introduced initially range from 0 to 3 rad, due to the fact it equivalents to having a maximum deviation of  $2\pi$ . The step considered is 1 rad, since it is a first approximation to the range of variances will be under study to get an error less than the specified. Figure 5.6 illustrates the results obtained.

**Figure 5.6** Influence of angle variance in LoS environments





**Figure 5.7** Influence of angle variance in LoS environments, shorter range

As it can be seen in Figure 5.6, the accuracy obtained with TOA estimation technique is less than 0.1m for all the values of the azimuth and elevation variance. This is mainly because the TOA technique is independent to the value of the azimuth and elevation angle. Moreover, we have considered there is not any error in the time estimation, therefore the time estimated observables will match with the real time observables, resulting in a zero-error.

On the other hand, with the AOA method the error obtained is quite high and almost equal for the azimuth and elevation angles. This means both angles influence in the same way to the estimation. From the graph, we obtain that the distance error is roughly 50 m for variances from 1 to 3 rad, and 20 m for a variance of 1 rad. Therefore, in order to get accuracies less than 5m we have to focus the study in the range from 0 to 0.5 rad, or equivalently up to  $28.65^\circ$ . Repeating again the simulation with the new range, we obtain the results illustrated in Figure 5.7.

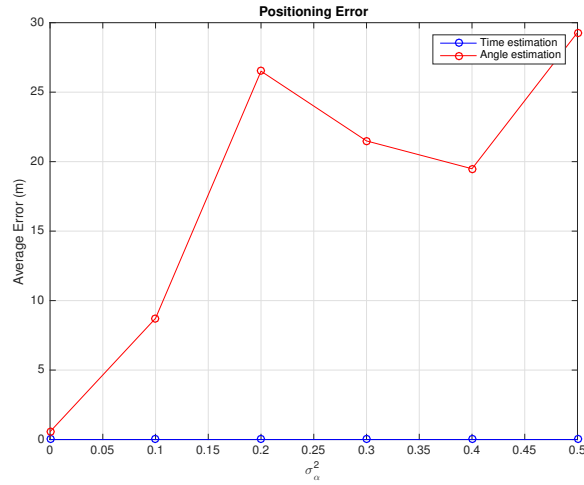
In Figure 5.7 with lower range of the azimuth and elevation variances we can see now that both angles do not affect equally to the estimation. The inaccuracies caused in the azimuth angle affect more significantly to the estimation error than the elevation angle, resulting in a higher error.

Looking at Equations 2.3–5.3 and Figure 2.4, with the azimuth estimation the two coordinates of the mobile,  $x$  and  $y$ , are obtained, whereas with the elevation estimation is obtained only the height information,  $z$ -coordinate. Thus, the noise added to the azimuth estimation angle will result in a higher error, due to the fact it is affecting to two coordinates simultaneously,  $x$  and  $y$ .

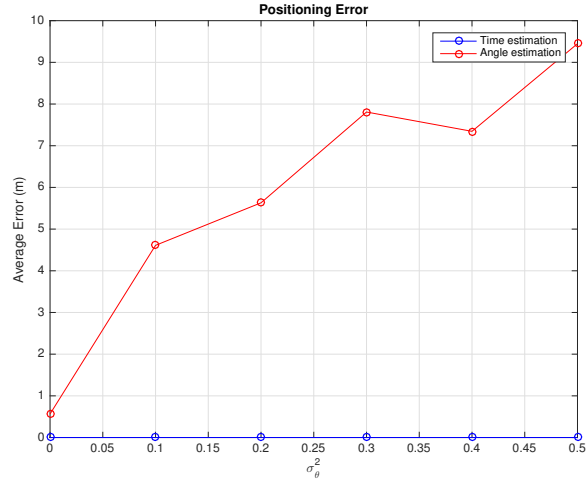
In Figures 5.8 and 5.9 the error introduced by each angle is represented the error introduced by each angle individually. As it can be seen, in presence of the same noise, the error introduced by the azimuth angle estimation is roughly 3 times greater than the error introduced by the elevation angle estimation.

Nevertheless, the error obtained in this shorter range remains higher than the objective, at least less than 5 m. Therefore the range of variances to study must be reducing again up to 0.15 rad for the elevation angle estimation and from 0 to 0.06 rad for the azimuth angle, approximately, resulting the plots represented in Figures 5.10 and 5.11.

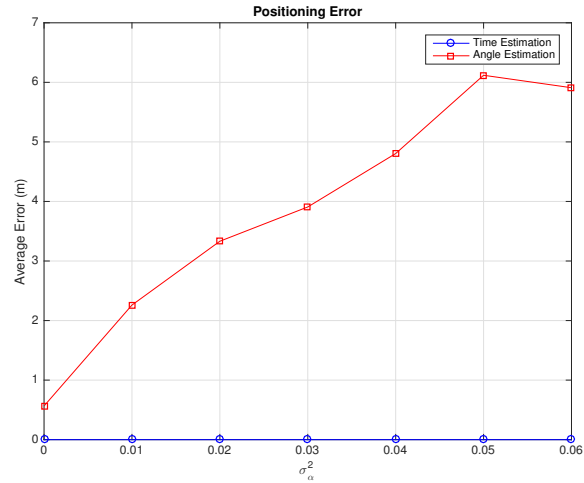
Thereby, in the new applicable range is possible to obtain the maximum allowed values of the azimuth and elevation variances in order to get the accuracies specified, as it is represented in Table 5.2.



**Figure 5.8** Influence of azimuth variance in LoS environments



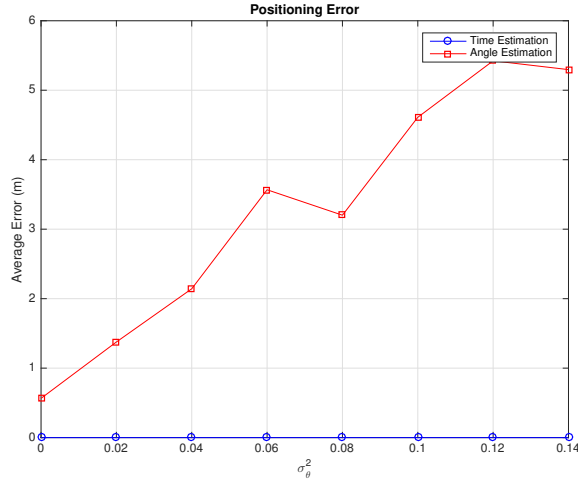
**Figure 5.9** Influence of elevation variance in LoS environments



**Figure 5.10** Influence of azimuth variance in LoS environments, shorter range

**Table 5.2** Maximum allowed azimuth and elevation variance in LoS environments

Target maximum positioning error	Azimuth Variance $\sigma_\alpha^2(rad)$	Azimuth Variance $\sigma_\alpha^2(deg)$	Elevation Variance $\sigma_\theta^2(rad)$	Elevation Variance $\sigma_\theta^2(deg)$
5 m	0.0375 rad	2.1486°	0.11 rad	6.3025°
1 m	0.0015 rad	0.0859°	0.006 rad	0.3438°
0.1 m	—	—	—	—



**Figure 5.11** Influence of elevation variance in LoS environments, shorter range

### 5.2.2 Influence of NLoS environments

In this section we have studied the influence of NLoS environments in the estimation error with the algorithms proposed for the IEEE 802.11az standard, TOA and AOA.

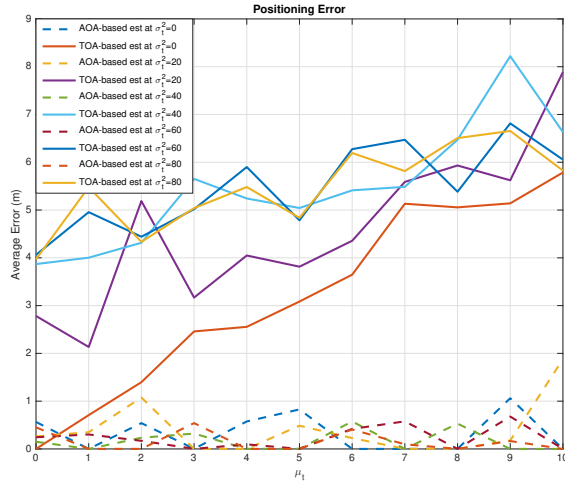
NLoS scenarios have been emulated by adding a noise with mean values different than zero to the time and angle estimators. In this way, the mean value of the noise will represent the multipaths and fading produced in the communication.

Following with the same idea as in Section 5.2.1, we have studied the influence of the variance and the mean value of the noise over the different estimators, time and angle. In the same way as before, we have divided this last case into the study of the influence of azimuth and elevation angles.

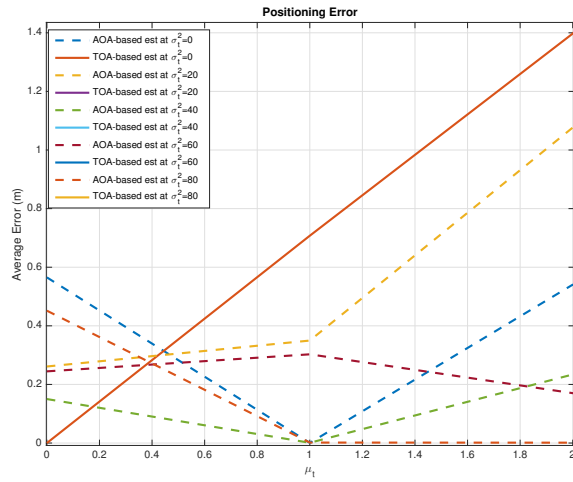
#### Influence of a bias in time observables

Assuming there is not any noise affecting to the angle estimation, we have studied the influence of the mean value in the estimation for some values of the time variance. The values of the time variance considered in this study take advantage of the results presented in Section 5.2.1, in which we have determined the maximum allowed value of the time variance in order to get accuracies less than 5 m is 80 ns. Therefore, we have studied the influence of the NLoS environment for time variance values from 0

to 80 ns, obtained from Section 5.2.1.



**Figure 5.12** Influence of mean value of noise in time estimation  $\mu_t$ , NLoS environments



**Figure 5.13** Influence of mean value of noise in time estimation, NLoS environments  $\mu_t$ , shorter range

Initially, the study has been done considering a large range of mean values, from 0 to 10 ns, since firstly we have to obtain the shape of the resulting distance error, as it is represented in Figure 5.12. The shape of the distance error as a function of the mean value of the noise suits to our expectations: the greater the value of the mean value the higher the error will be, due to the fact number of multipaths produced is higher.

Shortening the mean value range of the noise, in order to determine the exact values of the mean value to which the error is less than 1 and 0.1 m, we obtain the graph that is illustrated in Figure 5.13. As it can be seen, the error obtained is not linearly increasing as the value of the mean value of the noise increases, but varies a bit.

However it is possible to determine the maximum range of the mean value for each of simulated time variances, in order to reach the accuracies specified, of 5, 1 and 0.1. These values are summarized in Table 5.3. Accuracies less than 1 m or 0.1m are only possible whether the time variance is zero, this means in absence of noise.

**Table 5.3** Maximum allowed mean value of noise in time estimation, NLoS environments

Target maximum positioning error	$\mu_t(\sigma_t^2 = 0ns)$	$\mu_t(\sigma_t^2 = 20ns)$	$\mu_t(\sigma_t^2 = 40ns)$	$\mu_t(\sigma_t^2 = 60ns)$	$\mu_t(\sigma_t^2 = 80ns)$
5 m	8 ns	6.5 ns	2.5 ns	3 ns	0.7 ns
1 m	1.42 ns	—	—	—	—
0.1 m	—	—	—	—	—

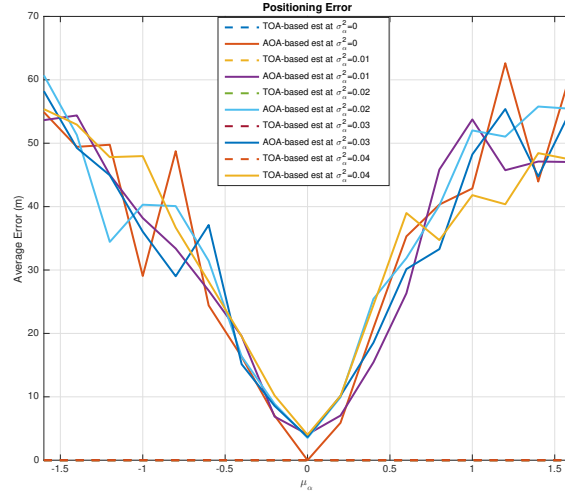
### Influence of a bias in angle observables

In this part we will characterize the influence of the mean value of the noise in the azimuth and elevation angles estimation, assuming in both cases the time estimation is not affected by any noise and in LoS. The study will be done for the azimuth and elevation variances values obtained in Section 5.2.1, up to 0.0375 and 0.11 rad, respectively.

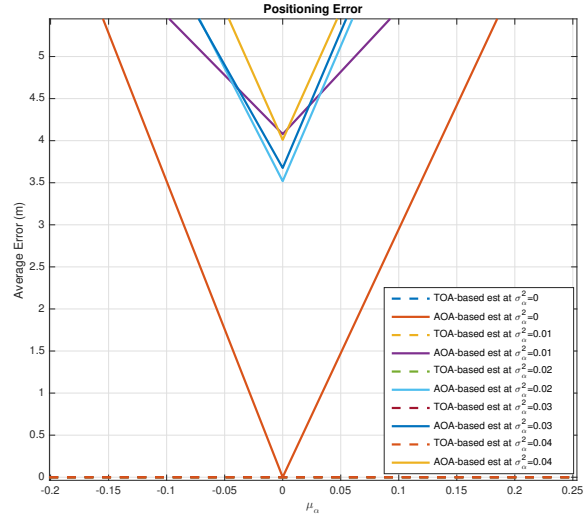
- **Azimuth angle estimation**

Assuming the noise is only affecting to the azimuth angle, i.e. the elevation variance is zero, we study the error distribution as a function of the mean value of the noise in azimuth estimation, for some azimuth variance values which accuracy is less than 5 m. In Figure 5.14 is represented the above-mentioned idea, where the range of the mean values considered is from  $-\pi$  to  $\pi$  rad.

The error obtained is almost symmetrical respect to zero mean value of the noise, and it increase rapidly as the mean value does. The range in which the accuracy is



**Figure 5.14** Influence of mean value of noise in azimuth estimation  $\mu_\alpha$ , NLoS environments. TOA estimates are always zero.



**Figure 5.15** Influence of mean value of noise in azimuth estimation  $\mu_\alpha$ , NLoS environments, shorter range. TOA estimates are always zero.

less than is between -0.2 and 0.2 rad, which equivalents, roughly, to  $34^\circ$ . In Figure 5.15 is represented the error obtained in this range. From this figure, we can see that errors less than 1 or 0.1 are only possible whether the azimuth variance is 0, i.e. no noise. The rest of the azimuth variances will remain an error of at least of 3.5 m in the estimates position.

Table 5.4 shows the maximum allowed range for the mean value of noise according to the objective accuracy.

**Table 5.4** Maximum allowed mean value of noise in azimuth estimation, NLoS environments

Error	$\mu_\alpha(\sigma_\alpha^2 = 0rad)$	$\mu_\alpha(\sigma_\alpha^2 = 0.01rad)$	$\mu_\alpha(\sigma_\alpha^2 = 0.02rad)$	$\mu_\alpha(\sigma_\alpha^2 = 0.03rad)$	$\mu_\alpha(\sigma_\alpha^2 = 0.04rad)$
5 m	0.31 rad (17.7617°)	0.12 rad (6.8755°)	0.11 rad (6.3025°)	0.105 rad (6.0161°)	0.06 rad (3.4377°)
1 m	0.063 rad (3.6096°)	—	—	—	—
0.1 m	0.005 rad (0.2865°)	—	—	—	—

- **Elevation angle estimation**

Repeating the same process as in azimuth angle estimation, for the elevation variances obtained in Section 5.2.1, we obtain the results represented in Figure 5.16.

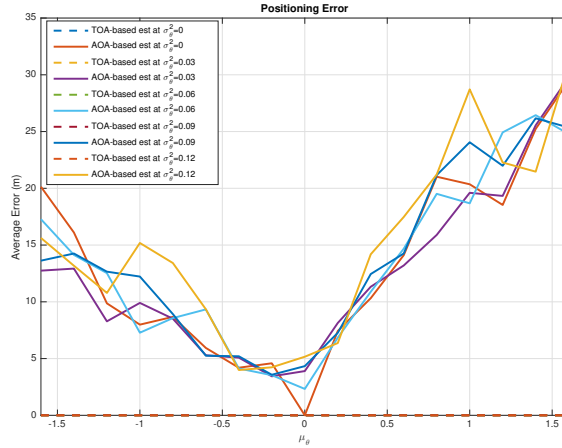
On the contrary than the azimuth angle estimation, the distribution of the error is not symmetric. The positive values of the mean value of the noise affect more significantly to the estimation causing higher errors than the negative values. However, the results of the positioning error for the elevation angle influence are better than the obtained for the azimuth angle influence, as it is shown in Figures 5.16 and 5.14. The maximum allowed range of mean values is greater than the obtained for the azimuth angle influence

In order to determine the maximum range of the mean values of the noise for which the error is below 5, 1 and 0.1m, we reduce the range of values considered in the simulation up to 0.3 rad. The resulting distribution of the error is represented

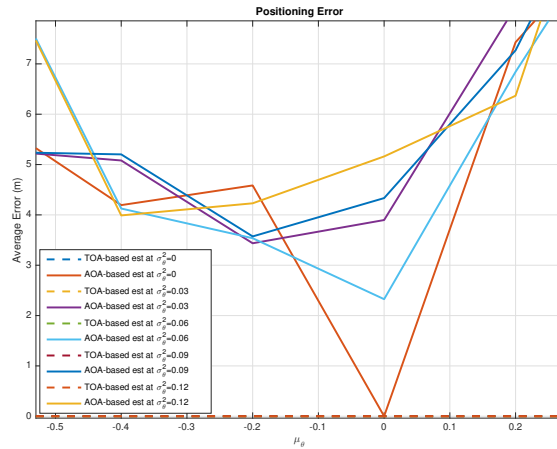


in Figure 5.17, where it can be seen, as in the previous case, accuracies of 1 and 0.1 m are only possible whether the elevation variance is 0.

In Table 5.5 are summarized the maximum allowed mean values for each value of the elevation variance considered, in order to achieve the objective accuracies.



**Figure 5.16** Influence of mean value of noise in elevation estimation  $\mu_\theta$ , NLoS environments. TOA estimates are always zero.



**Figure 5.17** Influence of mean value of noise in elevation estimation  $\mu_\theta$ , NLoS environments, shorter range. TOA estimates are always zero.

**Table 5.5** Maximum allowed mean value of noise in elevation estimation  $\mu_\alpha$ , NLoS environments

Target maximum position-error	$\mu_\theta(\sigma_\theta^2 = 0rad)$	$\mu_\theta(\sigma_\theta^2 = 0.03rad)$	$\mu_\theta(\sigma_\theta^2 = 0.06rad)$	$\mu_\theta(\sigma_\theta^2 = 0.09rad)$	$\mu_\theta(\sigma_\theta^2 = 0.12rad)$
5 m	0.64 rad (36.6693°)	0.49 rad (28.0749°)	0.45 rad (25.7831°)	0.41 rad (23.4913°)	0.39 rad (22.3454°)
1 m	0.07 rad (4.0107°)	—	—	—	—
0.1 m	0.007 rad (0.4011°)	—	—	—	—

### 5.2.3 Influence of the number of APs

In order to evaluate the influence of the number of APs in the estimation process, we have taken into consideration the results obtained in Sections 5.2.1 and 5.2.2. Therefore, assuming as a basis the values for the time, azimuth and elevation variances and mean values that allow to achieve accuracies less than 5 m for 10 AP, we have extended the study and analysed how is the error obtained for that variances when the number of APs considered is greater or smaller than 10.

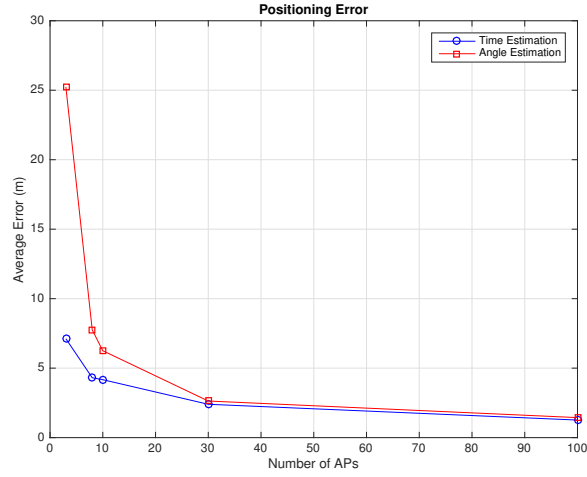
In Table 5.6 we summarize the values for the time and angle estimation considered in the simulation, according to the type scenario LoS or NLoS.

**Table 5.6** Influence of number of APs set-up values

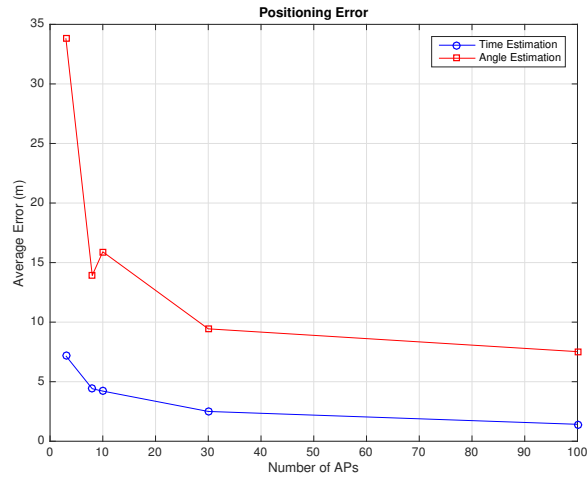
Scenario	$\sigma_t^2$	$\sigma_\alpha^2$	$\sigma_\theta^2$	$\mu_t$	$\mu_\alpha$	$\mu_\theta$
LoS	80 ns	0.0375 rad	0.11 rad	N/A	N/A	N/A
NLoS	80 ns	0.0375 rad	0.11 rad	0.7 ns	0.06 rad	0.39 rad

The obtained results are illustrated in Figures 5.18 and 5.19, for the LoS and NLoS environment considered, respectively.

From Figures 5.18 and 5.19, we can notice that the error with AOA method for 10 AP overpass, slightly, the 5 m. This is because the values of the azimuth and elevation variance and mean introduced have been determined when the other parameter was zero, i.e. the azimuth variance of 0.0375 ns in LoS environment was determined making the elevation variance equals to 0. Therefore, the error is slightly



**Figure 5.18** Influence of the number of APs, LoS environments



**Figure 5.19** Influence of the number of APs, NLoS environments

higher than the one studied in Sections 5.2.1 and 5.2.2.

In general, as we increase the number of APs considered in the scenario, the accuracy in the estimation process obtained increases. In Figures 5.18 and 5.19 we can see, effectively the error decreases as the number of APs increases. However, it increases drastically when instead of considering 3 APs, the minimum allowed in the positioning methods implemented. The error continues to decrease with an increase number of APs, but after 10 APs the decrease is slower than up to 10 APs.

As it can be observed from Figures 5.18 and 5.19, the estimation error obtained in NLoS environment is higher than in LoS, since the multipaths beside the noise effects affect significantly to the estimation under NLoS environments.

Moreover, it can be noticed the difference in accuracy between the TOA and AOA methods, being better when TOA method is used. In LoS plot, both methods TOA and AOA end converging if more than 30 APs are considered. On the contrary, in NLoS situation the both methods TOA and AOA decrease parallel but they do not converge in any moment.

#### 5.2.4 Joint Time and Angle estimation proposed method

In order to get the objective accuracies (5, 1 and 0.1m) in those cases where are not possible, mainly with AOA method, a joint estimation method using TOA and AOA measurements has been proposed, implemented and tested. The methodology of the method is based on the explained in Section 5.1.1:

1. Calculating jointly the time and angle observables. With the TOA and AOA measurements, we will construct a vector with the joint observables, as it is indicated in Equation 5.17:

$$y_{obs} = [t \ \alpha \ \theta] \quad (5.17)$$

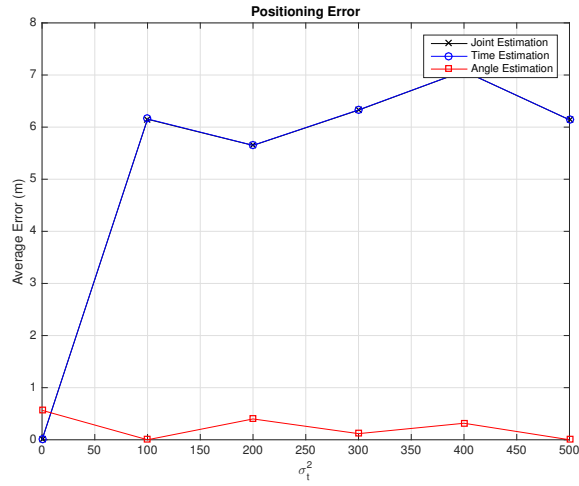
The first  $N_{AP}$  values will be correspond with the time estimation, the following  $N_{AP}$  values will be the alpha observables, and the last  $N_{AP}$  values will be the theta values. This vector will be the input and reference data for the optimization process.

2. Making use of the equations 4.9, 2.3, 5.3 from the TOA and AOA methods, we will estimate jointly the time and angle observables, constructing an objective function to minimize such as:

$$f = \begin{bmatrix} t_{est_1} \\ t_{est_2} \\ \dots \\ t_{est_i} \\ \alpha_{est_1} \\ \alpha_{est_2} \\ \dots \\ \alpha_{est_i} \\ \theta_{est_1} \\ \theta_{est_2} \\ \dots \\ \theta_{est_i} \end{bmatrix} \quad (5.18)$$

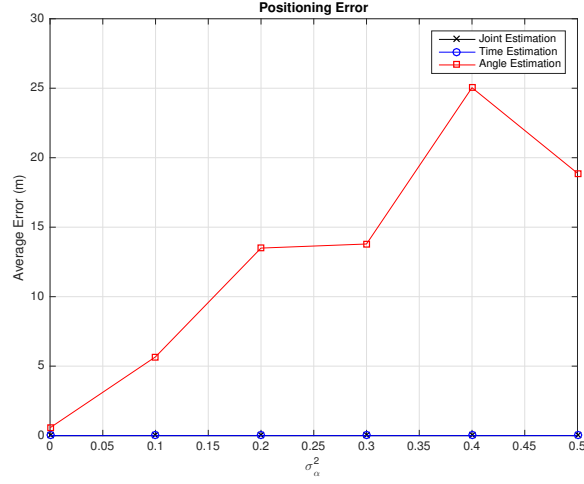
3. Using lsqcurvefit and multistart methods, we will try to minimize the objective function and obtain the estimated position of the mobile.

In order to test the features of the proposed method as well as the advantages/disadvantages respect to TOA and AOA, we have run the same simulations as in Section 5.2.1 for LoS environments affected by noise, obtaining the results showed in Figures 5.20, 5.21 and 5.22.

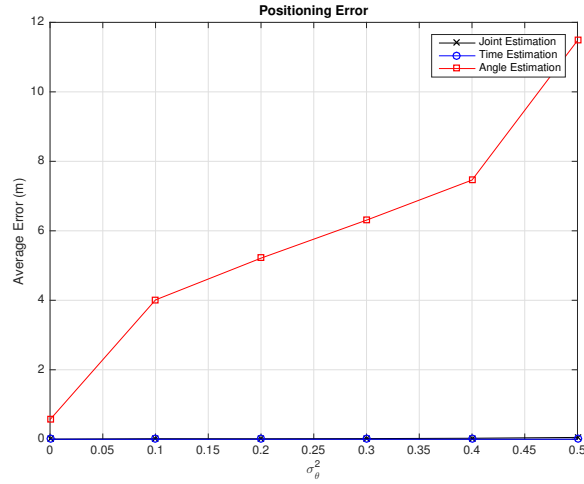


**Figure 5.20** Joint estimation illustration, Time Variance

The joint estimation follows the curve of the time estimation, even although the error is greater than the obtained with AOA estimation shown in Figure 5.21 . Doing



**Figure 5.21** Joint estimation illustration, Azimuth Variance

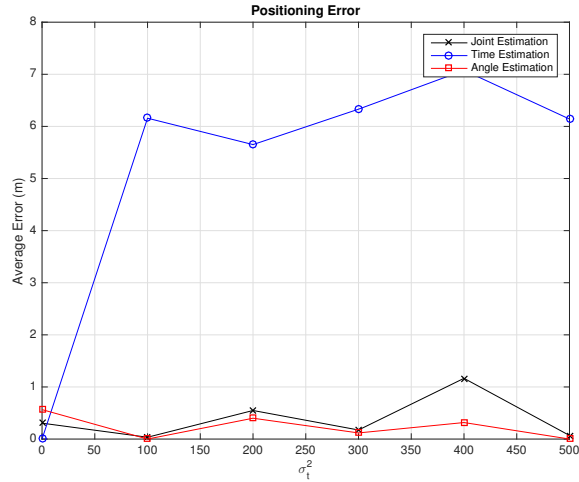


**Figure 5.22** Joint estimation illustration, Elevation Variance

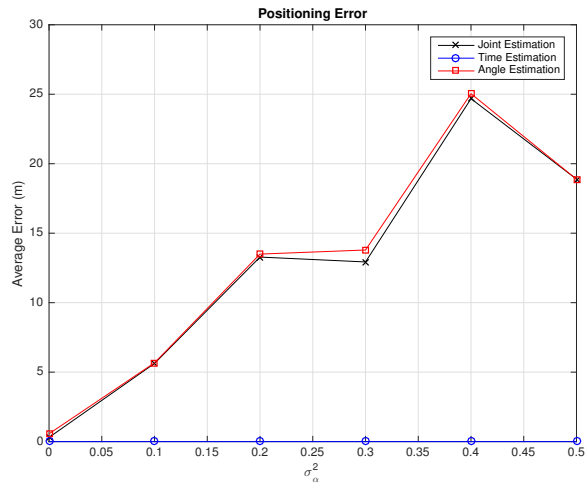
some research in the joint estimation process, we discovered that the estimated time and angle observables had different weights in the lsqcurvefit function, therefore their influence in the minimization process will be different. The weight of the time estimators is higher than the angle estimators, thus the curves of the joint estimation follow the TOA method.

The solution of his problem consists of finding out the right weights to apply to the time and angle estimators. However, it is not trivial because the objective function is a vector of  $(3 \times N_{AP}) \times 1$ , in which the time, azimuth and angle estimators are

independent and there is not any equation which relation them. Therefore, knowing that the time estimators present a higher influence in the joint estimation, we tried to apply a normalization factor to these estimators. Due to the fact, the value of the time estimators obtained are in the order of power of 2 and the azimuth and angle estimators are in the order of power of -1, we have applied a normalization factor of 1000 to make comparable all the estimator, obtaining the results showed in Figures 5.23, 5.24 and 5.25.

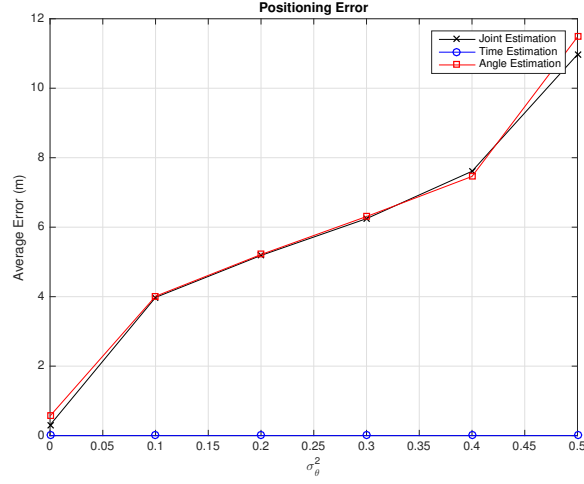


**Figure 5.23** Joint estimation illustration 2, Time Variance



**Figure 5.24** Joint estimation illustration 2, Azimuth Variance

In this case, the joint estimation follows the curve of AOA, although the error with TOA is smaller. Trying with other normalization values the results obtained were



**Figure 5.25** Joint estimation illustration 2, Elevation Variance

the same. These tests indicate that the joint estimation method proposed in this thesis will never be in the middle of the TOA and AOA curves, it will be always follow one of the curve.

Our tests on joint TOA-AOA shows that a simple linear combining of estimators is not helping in achieving better results than individual TOA or AOA estimates. This also points out towards the need of using more advanced joint estimators, such as Extended Kalman Filter (EKF) or Unscented Kalman Filter (UKF), which have not been studied in this thesis and remain a topic of future investigations.



## 6. CONCLUSIONS AND FUTURE WORK

Along the thesis work, we evaluated the performance of the proposed passive positioning technique based on a modified TOA algorithm and the AOA algorithm, under different assumptions: LoS and NLoS environments. Thus, we discovered that the modified TOA technique allows to achieve, in general, higher accuracy than the tradition AOA method. Therefore, AOA standalone method will not fulfil the requirements of IEEE 802.11az which specifies accuracies in the positioning less than 0.1 m for the NGP.

The number of the APs considered in the scenarios under study has been also investigated. It was observed that the positioning accuracy improves as the number of the APs increases, since the device has more information to determine its position. For 3 APs that is the minimum value of APs is required to determine univocally a point, we proved the error is quite high, at least of 25 m with AOA, the fourth part of the size of the building (100m), and 8 m with TOA algorithm.

Moreover, we created and studied a joint estimation technique in order to evaluate whether the performance of the AOA technique, mainly, improved and the estimation error decreased in order to fulfil IEEE 802.11az requirements. However, we have seen that the joint estimation technique does not provide the results we expected. With the joint estimation the obtained error should be better than with the TOA and AOA methods separately, because it is based on the estimation of three parameters: time, azimuth and elevation. Therefore, with this information it should be able to position the device with higher accuracy than TOA and AOA or at least, with the same accuracy of the best method. However, we saw that the joint estimation does not fit with the `lsqcurvefit` function used in the thesis work to allow the positioning and thus it is likely that non-linear estimation approaches such as EKF are needed for a performant joint estimation.

As a future work, a research about the joint estimation technique using other functions different than `lsqcurvefit` during the optimization problem should be done,

such as EKF filters. In addition, a research of a new technique based on a modified AOA method could be done, in order to reach the accuracies specified in the IEEE 802.11az standard.

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